JAM (2020-2021)
Mock Test No: 2
Subject: Mathematics for Chemistry

Date: 02/02/2020 **Time:** 1 hour 30 mins **Full Marks: 50**

No. of questions attempted	No. of correct answers	No. of wrong answers	Marks obtained	Total

- There is only one correct option.
- Tick (✓) to the correct option.
- There is a negative marking of 0.25 for each wrong attempt.

1. **Which of the following function is neither even nor odd?**
   (a) $\sin x$ (b) $\cos x$ (c) e^x (d) $\frac{e^x+e^{-x}}{2}$.

2. **Which of the following function is not single-valued?**
   (a) $\sin x$ (b) x^2 (c) $\sqrt{x}$ (d) e^{-x}.

3. **Which of the following function is continuous but not differentiable within $-\infty \leq x \leq \infty$?**
   (a) $|x|$ (b) x^2 (c) $\sqrt{x}$ (d) e^{-x}.

4. $\lim_{x\to 0} \frac{\sin x}{x}$ **is**
   (a) 0 (b) 1 (c) ∞ (d) $-\infty$

5. **Which of the following is a transcendental equation in x?**
   (a) $x - xy^2 + 5 = 0$ (b) $(x+1)^2 + y^2 = 0$
   (c) $\sqrt{x} + \sqrt{y} = \sqrt{c}$ (d) $\tan x = e^x$.

6. **The local maxima of $f(x) = x^2(1-x)^2$ appear at**
   (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) -1

7. **The value of the integral $\int_0^{\pi} \cos^2\theta \sin\theta d\theta$ is**
   (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) π (d) 0

8. **The value of the integral** $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-x}dx$ **is**

   (a) $\frac{\sqrt{\pi}}{2}$ (b) 0 (c) π (d) 1

9. **The value of the series** $\sum_{n=1}^{\infty}\frac{1}{n^4}$ **is**

   (a) $\frac{\pi^4}{90}$ (b) $\frac{\pi^4}{15}$ (c) π (d) ∞

10. **Which of the following is not a general solution of the differential equation** $\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}+\omega^2x(t)=0$

    (a) $x(t)=A\sin\omega t$ (b) $x(t)=B\cos\omega t$

    (c) $x(t)=Ce^{i\omega t}$ (d) $x(t)=D\tan\omega t$

11. **The volume element in spherical coordinate system is given by**

    (a) $dV=r^2\sin\theta drd\theta d\phi$ (b) $dV=r\sin\theta drd\theta d\phi$

    (c) $dV=drd\theta d\phi$ (d) $dV=r^2\sin\theta\cos\phi drd\theta d\phi$

12. **Two vectors are given by** $\vec{A}=2\hat{i}-\hat{j}+3\hat{k}$ **and** $\vec{B}=-\hat{i}+2\hat{j}-\hat{k}$**. Then scalar product of** $\vec{A}$ **and** $\vec{B}$ **is**

    (a) $-5\hat{i}-\hat{j}+6\hat{k}$ (b) -7

    (c) $\hat{i}-3\hat{j}+4\hat{k}$ (d) 19

13. **The inverse of the matrix** $\begin{bmatrix}1&-1&0\\0&1&1\\2&2&0\end{bmatrix}$ **is**

    (a) $\begin{bmatrix}\frac{1}{2}&0&\frac{1}{4}\\-\frac{1}{2}&0&\frac{1}{4}\\\frac{1}{2}&1&\frac{1}{4}\end{bmatrix}$ (b) $\begin{bmatrix}1&-1&1\\5&1&1\\2&2&0\end{bmatrix}$

    (c) $\begin{bmatrix}1&1&0\\5&1&1\\2&2&0\end{bmatrix}$ (d) $\begin{bmatrix}1&5&0\\3&1&1\\2&2&7\end{bmatrix}$

14. **The value of the determinant** $\begin{vmatrix}1&5&7\\0&0&0\\2&2&7\end{vmatrix}$ **is**

    (a) 5 (b) 0 (c) -12 (d) 7

15. **Using Striling approximation the value of** $\ln 10!$ **is (approximately)**

    (a) 15.092 (b) 24.035 (c) 10.021 (d) 15.456

16. **The simplest normalized continuous distribution is given as** $p(x)=A$ **when** $a\le x\le b$ **and** 0 **otherwise. The value of** A **is**

    (a) $\frac{1}{b-a}$ (b) $\frac{1}{b+a}$ (c) $\frac{1}{b^2-a^2}$ (d) $\frac{1}{b^3-a^3}$

17. **The Gaussian distribution is given as** $p(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\dfrac{x^2}{2\sigma^2}}$ **when** $-\infty\le x\le\infty$**. The variance in** x **is**

    (a) σ (b) $\sqrt{\sigma}$ (c) σ^2 (d) 0

18. **If A and B are two nonsingular square matrix then $(AB)^{-1}$ is given by**
(a) $A^{-1}B^{-1}$ (b) $B^{-1}A^{-1}$ (c) $A^{-1}B$ (d) AB^{-1}

19. **The Laplacian operator in circular polar coordinate system is given as**
(a) $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ (b) $\nabla^2 = \frac{\partial^2}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$
(c) $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial \theta^2}$ (d) $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$

20. **Two complex numbers are given as $Z_1 = 2 + 3i$ and $Z_2 = 4 + 6i$. Then**
(a) $Z_1 > Z_2$ (b) $Z_2 > Z_1$ (c) $Z_1 = Z_2$ (d) cannot be said.

21. **Which of the following is an irrational number?**
(a) 1.5 (b) $\frac{5}{2}$ (c) -3 (d) π

22. **For van der Waals gas, at critical point**
(a) $\frac{\partial P}{\partial V} = 0$ and $\frac{\partial^2 P}{\partial V^2} = 0$ (b) $\frac{\partial P}{\partial V} = 0$ and $\frac{\partial^2 P}{\partial V^2} > 0$
(c) $\frac{\partial P}{\partial V} = 0$ and $\frac{\partial^2 P}{\partial V^2} < 0$ (d) $\frac{\partial P}{\partial V} > 0$ and $\frac{\partial^2 P}{\partial V^2} < 0$

23. **The value of the integral $\int_{-\infty}^{+\infty} 2\sin x \cos x dx$ is**
(a) 1 (b) 0 (c) π (d) 2π

24. **The value of $(\cos\pi)^i$ is**
(a) $e^{-\pi}$ (b) 0 (c) e^{π} (d) -1

25. **For a complex number Z, $Z^* = -Z$, then Z is**
(a) real (b) purely imaginary
(c) must be negative (d) must be nonzero.

26. **Which of the following has a nonterminating decimal expansion?**
(a) $\frac{77}{210}$ (b) $\frac{23}{8}$ (c) $\frac{17}{8}$ (d) $\frac{35}{50}$

27. **The HCF $\times$ LCM for the numbers 50 and 20 is**
(a) 10 (b) 100 (c) 1000 (d) 50

28. **If $f(x) = \ln x^3$, then $f''(3)$ is**
(a) $-\frac{1}{3}$ (b) -1 (c) -3 (d) 1 (e) none of these

29. **In a spherical polar coordinate system, a point A at (x, y, z) in the Cartesian coordinate system can be described by (r, θ, ϕ) where r, θ, and ϕ have their usual meaning. Expression for the volume of an infinitesimally small cube confined by dx, dy, and dz in terms of the spherical coordinate system is given by**
(a) $drd\theta d\phi$ (b) $r\sin\theta drd\theta d\phi$
(c) $r^2\sin^2\theta drd\theta d\phi$ (b) $r^2\sin\theta drd\theta d\phi$

30. **What is the value of $i\log_{10} i^2$, where $i = \sqrt{-1}$?**
(a) Real and positive number
(b) Real and negative number
(c) Complex number
(d) Purely imaginary

31. **The integral** $\int_{-a}^{a} \cos(x)\sin(x)dx$

(a) Equals to zero for any value of a, and $\cos(x)$ is symmetric in the range of the integral.
(b) Is not equal to zero except for certain values of a, and $\sin(x)$ is antisymmetric in the range of the integral.
(c) Is not equal to zero except for certain values of a, and $\cos(x)$ is symmetric in the range of the integral.
(d) Has a non-zero value depending on a.

32. **What is the value of** i^{i+2}**, where** $i = \sqrt{-1}$**?**

(a) Real number (b) Complex number
(c) Cannot be calculated (d) None of the above.

33. **Consider the trigonometric function** $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$**. It can be simplified as**

(a) $\frac{1+\sec A - \tan A}{1 - \sec A + \tan A}$ (b) $\csc A + \cot A$
(c) $\frac{1}{\csc A - \cot A}$ (d) All of the above

34. **What is** i**? (Given** $i^2 = -1$**)**

(a) a real number (b) a complex number
(c) an imaginary number (d) none of the above

35. **The roots of the equation** $x^3 + ax^2 - bx + c = 0$ **are three consecutive integers. What is the maximum value of** b**?**

(a) -2 (b) 0 (c) 1 (d) 2

36. **Consider the following two infinite series :**

$$A = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad B = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

where x **is real. What is the value of** $A^2 + B^2$**?**

(a) 0 (b) ∞
(c) 1 (d) Its value cannot be defined, as the series A and B are divergent

37. **The number of vectors of unit length perpendicular to vectors** $\vec{a} = (1, 1, 0)$ **and** $\vec{b} = (0, 1, 1)$ **is**

(a) one (b) two (c) three (d) infinite (e) none of these

38. **Find** $\frac{dy}{dx}$ **when** $y = \log_2 x$**.**

(a) $x^2(1 + 3\log x)$ (b) $x(1 + 3\log x)$
(c) $x^4(1 + 8\log x)$ (d) $x^2(1 + 5\log x)$

39. **An electron tunnels through a square barrier of width** d **and height** h**. If** h **is fixed and** d **is normally distributed, then the electron transfer probability distribution is**

(a) a normal distribution (b) a delta function
(c) a log normal distribution (d) a poisson distribution

40. **Out of the following the only quantity that is a vector is**
(a) specific heat (b) temperature
(c) torque (d) speed

41. **Two vectors $\vec{A}$ and $\vec{B}$ are perpendicular when**
(a) $\vec{A}.\vec{B} = 0$ (b) $\vec{A} \times \vec{B} = 0$
(c) $\vec{A} = 0$ (d) $\vec{B} = 0$

42. **If $\int x \sin x dx = -x \cos x + \alpha$, then the value of α is**
(a) $\sin x + c$ (b) $\cos x + c$
(c) $x \cos x + c$ (d) $\cos x - \sin x + c$

43. **The value of $\lim_{x \to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}}$ is**
(a) e^2 (b) e^3 (c) e^5 (d) none of these

44. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ **is**
(a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

45. $\lim_{x \to \infty} \left(\frac{x}{2+x}\right)^{2x}$ **is equal to**
(a) e^{-4} (b) e^{-6} (c) e^{-2} (d) none of these

46. **Function written as $y = f(x) = a_0 + a_1 x$ is general form of**
(a) linear function (b) variable function
(c) constant function (d) parabolic function.

47. **If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then when $t = 1$, $\frac{d^2y}{dx^2}$ is**
(a) 1 (b) -1 (c) 0 (d) 3 (e) $\frac{1}{2}$

48. **For any vector $\vec{a}$, the value of $\left(\vec{a} \times \hat{i}\right)^2 + \left(\vec{a} \times \hat{k}\right)^2$ is equal to**
(a) $3\vec{a^2}$ (b) $\vec{a^2}$ (c) $2\vec{a^2}$ (d) $4\vec{a^2}$

49. **The points with position vectors $60i + 3j$, $40i - 8j$, $ai - 52j$ are collinear if**
(a) $a = -40$ (b) $a = 40$ (c) $a = 20$ (d) none of these

50. **If the area bounded by the $x-$axis, the curve $y = f(x)$ and the lines $x = 1$, $x = b$ is equal to $\sqrt{b^2+1} - \sqrt{2}$ for all $b > 1$, then $f(x)$ is**
(a) $\sqrt{x-1}$ (b) $\sqrt{x-1}$ (c) $\sqrt{x^2+1}$ (d) $\frac{x}{\sqrt{1+x^2}}$