# Jhargram Raj College

Assignment-1
$3^{rd}$ semester Physics Honours
Paper : **CC-5**

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✍ Kronecker's method of integration.

$$\int g(x)f(x)dx = g(x)F_1(x) - g'(x)F_2(x) + g''(x)F_3(x) + \cdots \quad (1)$$

1. Sketch the periodic extension of $f(t) = t/\pi$, $-\pi < t < \pi$. Find its Fourier series.

2. Sketch the periodic extension of $f(t) = 0$ for $t < 0$, $f(t) = 1$ for $t > 0$, if the funcdamental interval is $(-1, 1)$.

3. A function $f(x)$ is defined only over the range $0 < t < 4$ as

$$f(x) = \begin{cases} t, & 0 < t < 2 \\ 4 - t & 2 < t < 4 \end{cases} \quad (2)$$

Find the half range cosine and sine expansion of $f(x)$.

4. Obtain a Fourier series to represent the function

$$f(x) = |x| \quad for \quad -\pi < x < \pi \quad (3)$$

and hence deduce $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

5. A periodic square wave has a period 4. The function generating the square is

$$f(t) = \begin{cases} 0 & for \quad -2 < t < -1 \\ k & for \quad -1 < t < 1 \\ 0 & for \quad 1 < t < 2 \end{cases} \quad (4)$$

Find the Fourier series of the function.

6. If the Fourier series for $f(x)$ converges uniformly in $(-l, l)$, then show that

$$\frac{1}{l}\int_l^l \{f(x)\}^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

where $a_0, a_n, b_n$ are the Fourier's constants.

7. Find the Fourier series of the function $f(x)$in the interval $-\pi < x < \pi$, where

$$f(x) = \begin{cases} 0 & when \quad -\pi < x \leq 0 \\ \dfrac{\pi x}{4} & when \quad 0 < x < \pi \end{cases}$$

and hence show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

8. Show that the function $f(x) = x^3 - \pi^2 x$ has the Fourier series

$$f(x) = \sum_{n=1}^{\infty} \frac{12(-1)^n}{n^3} \sin nx \quad (5)$$

Now show that $\sum_1^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$