Jhargram Raj College

Problem Set - Small Oscillation

Code: Sem_5_Assignment_1

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-by S.S

1. **Couple Pendulum**

   If you have to oscillating pendulum, interacting with each other via some kind of a spring system, this kind of a system is known as "Couple Pendulum".

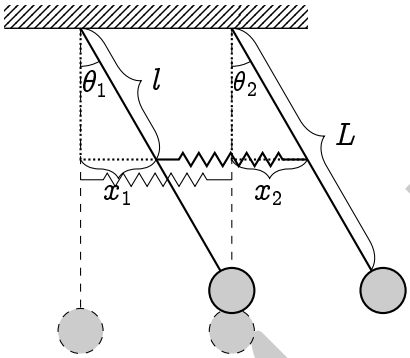

Figure 1: Coupled Pendulum

   (a) Using small oscillation approximation show that the Lagrangian of the system (assume the masses are equal) is given by

$$L = \frac{1}{2}ml^2\left(\dot{\theta}_1^2 + \dot{\theta}_2^2\right) - \frac{1}{2}mgl\left(\theta_1^2 + \theta_2^2\right) - \frac{1}{2}kl^2\left(\theta_2 - \theta_1\right)^2 + 2mgl$$

   (b) Find the equations of moton.

   (c) Find the $\mathbf{T}$ and $\mathbf{V}$ matrix.

   (d) Show that the normal frequencies are

$$\omega_1 = \sqrt{\frac{g}{l}};\ \omega_2 = \sqrt{\frac{g}{l} + \frac{2k}{m}}$$

   (e) Show that the general solution can be given b y

$$\begin{bmatrix}\theta_1(t)\\ \theta_2(t)\end{bmatrix} = A\begin{bmatrix}1\\1\end{bmatrix}\cos\omega t + B\begin{bmatrix}1\\-1\end{bmatrix}\sin\omega t \tag{1}$$

2. Consider two blocks and three springs as shown in the figure. Assume that all the motion are horizontal. When the blocks are at rest, all the springs are unstretched.

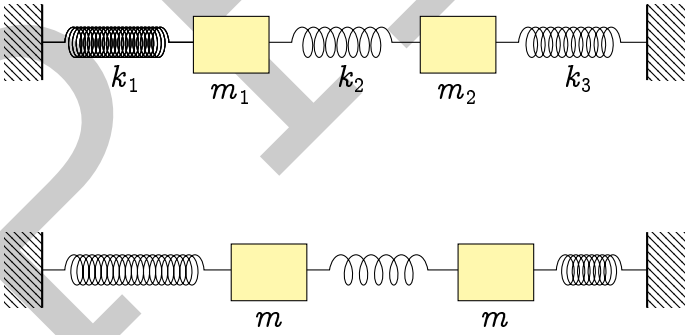

Figure 2: Horizontal motion of spring-mass system

   (a) Choose as generalized coordinates the displacement of each block from its equilibrium position, and write the Lagrangian.

   (b) Find the $\mathbf{T}$ and $\mathbf{V}$ matrices.

   (c) Suppose

$$\begin{cases} m_1 = 2m, & m_2 = m \\ k_1 = 4k, & k_2 = k, \quad k_3 = 2k \end{cases}$$

   Find the frequencies of small oscillations

   (d) Find the normal modes of the oscillation.

   (e) At time $t = 0$, mass m_1 is displaced by a distance b relative to its equilibrium position i.e $x_1(0) = b$. The other initial conditions are $x_2 = 0, \dot{x}_1(0) = 0$ and $\dot{x}_2(0) = 0$. Find $\tilde{t}$, the next time at which x_2 vanishes.