Jhargram Raj College

Problem Set - Fourier Series

Code: Sem_3_Assignment_1

Batch: Semester_3, 2021 – 22

-by S.S

Kronecker's Method of integration

$$\int g(x)f(x)\,dx = g(x)F_1(x) - g'(x)F_2(x) + g''(x)F_3(x) + \cdots \tag{1}$$

where,

$$F_1(x) = \int f(x)dx \tag{2}$$

$$F_2(x) = \int F_1(x)dx \tag{3}$$

$$\vdots \tag{4}$$

$$F_n(x) = \int F_{n-1}(x)dx \tag{5}$$

1. Find the Fourier Cosine Series for the function

$$f(x) = \begin{cases} 1 & 0 < x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \pi \end{cases}$$

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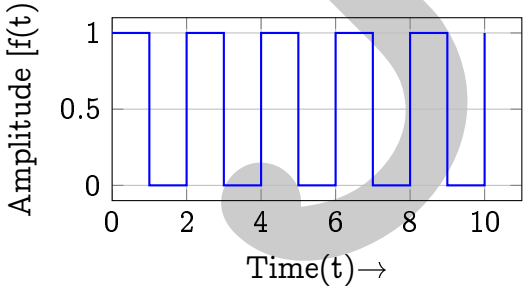
2. Consider the square wave as given below

Square Wave



(a) Write the functional form of the square wave function.

(b) Find the 'Fourier Series' for this square wave function.

3. Expand $f(x) = \sin x,\ 0 < x < \pi$, in a Fourier series and show that

$$f(x) = \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{\cos 2x}{2^2-1} + \frac{\cos 4x}{4^2-1} + \frac{\cos 6x}{6^2-1} + \cdots\right)$$

4. Expand $f(x) = x, 0 < x < 2$, in a half range series

(a) using sine series show that

$$x = \frac{4}{\pi}\left(\sin\frac{\pi x}{2} - \frac{1}{2}\sin\frac{2\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} - \cdots\right)$$

(b) Using cosine series show that

$$x = 1 - \frac{8}{\pi^2}\left(\cos\frac{\pi x}{2} + \frac{1}{3^2}\cos\frac{3\pi x}{2} + \frac{1}{5^2}\cos\frac{5\pi x}{2} + \cdots\right)$$

5. (a) Using the cosine series of the last problem [4(b)], write down the Parseval's identity corresponding to the Fourier series.

(b) From above show that

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots + \frac{1}{n^4} + \cdots = \frac{\pi^4}{90}$$

6. (a) Find a Fourier series for $f(x) = x^2, 0 < x < 2$, by integrating the sine series of $f(x) = x, 0 < x < 2$.

(b) Now show that

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$$