2018

CBCS

1st Semester

PHYSICS

PAPER—C1T

(Honours)

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Mathematical Physics

Group—A

Answer any *five* questions : 5×2

1. Solve : $\frac{dz}{dx} - xz = -x$.

(Turn Over)

2. Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C \rho^2 d\phi$ in case of polar coordinates (ρ, ϕ).

3. Prove that $\iint_S \hat{n}\, ds = \vec{0}$ for any closed surface S.

4. Two solutions of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ are e^x and xe^x. Is the general solution $y = c_1e^x + c_2xe^x$? Check by the Wronskian.

5. Rolling a dice three times evaluate the probability of having at least one six.

6. Determine the Jacobian for spherical polar co-ordinates.

(Continued)

7. Prove using the property of Dirac delta function

$\delta(x-a)=\delta(a-x)$

8. From a deck of 52 cards, two cards are drawn in succession. Find the chance that the first is a king and the second a queen if the first card in not replaced.

Group—B

Answer any *four* questions : 4×5

9. Give a rough plot of the force function $F(x)=x^2-4x+3$. What are the equilibrium points ? Are they stable or unstable and why ? 1+1+3

10. Initially at rest, a body of man m is falling under action of gravity and air resistance (R) proportional to the square of the velocity. (*i.e.* $R \propto v^2$)

Prove that $\frac{2kx}{m} = \log \frac{a^2}{a^2 - v^2}$

where v is the velocity achieved by the body after falling a distance x, g in the acceleration due to gravity and mg $= ka^2$. What is the maximum velocity the body can attain ? 5

11. (a) If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$ 3

(b) Evaluate : $\int_0^3 x^2 \delta(x+1) dx$ 2

12. Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$; $y(1) = 0$, $\frac{dy}{dx} = 1$. 5

13. (a) The Gaussian probability distribution is given by

$$P_G(x:\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\;; \quad -\infty<x<\infty$$ in usual

notation. Show that it has two points of inflexion at $x=\mu\pm\sigma$. 3

(b) Define the Dirac delta function $\delta(x)$. Mention the properties of $\delta(x)$. (1+1)

14. (a) What is Baye's theorem in the theory of Probability ? 2

(b) Consider three bags. The first one contains 3 white, 1 red, 2 green balls, the second one contains 2 white, 3 red, 1 green balls and the 3rd one contains 1 white, 2 red and 3 green balls. Two balls drawn out of a randomly chosen bag, are found to be one white and one red. Find the probability that the balls so drawn came from the second bag. 3

Group—C

Answer any *one* question : 1×10

15. (a) Find a unit vector perpendicular to the surface $(x-2)^2 + 5y^2 + 2z^2 = 8$ at the point $(1,1,1)$. 3

(b) If $\vec{A} = 6z\hat{i} + (2x+y)\hat{j} - x\hat{k}$, calculate $\iint_S \vec{A}\cdot\hat{n}\, ds$ over the entire surface S of the region bounded by the cylinder $x^2 + z^2 = 9$, $x = 0$, $y = 0$ and $y = 8$. 5

(c) An LIC agent sells on the average 3 insurance policies per week. Use Poisson probability distribution to calculate the probability that in a given week he will sell some policies. 2

16. (a) Using Lagrange's method of undetermined multiplier, find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ 4

(b) Solve the differential equation :

$$y''-2y'+y=e^x \log x$$ 4

(c) Find $\frac{du}{dt}$ if $u=\sin\left(\frac{x}{y}\right)$, $x = e^t$ and $y = t^2$. 2

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Mechanics

Group—A

Answer any *five* questions : 5×2

1. Distinguish between 'true' and 'fictitious' forces. 2

2. A machine gun fires 50 gm bullets at speed of 100 m/s. The gunner holding the machine gun in his hands can exert an average force of 180 newton against the gun. Determine the maximum number of bullets he can five per minute. 2

(Turn Over)

3. Calculate the work done by a force $F = kx^2$ acting on a particle at an angle 60° with x-axis to displace it from x_1 to x_2 along the x-axis. 2

4. The co-ordinate of a moving particle at any time 't' is given by $x = ct^2$ and $y = bt^2$. Find the speed of the particle. 2

5. Determine the Poissons's ratio and bulk modulus of a material, for which young's modulus is $1{\cdot}2\times10^5$ N/mm^2 and modulus of rigidity is $4{\cdot}8\times10^4$ N/mm^2.

6. Calculate the limiting velocity required by an earth's satellite for orbiting round the earth. $R = 6{\cdot}4\times10^6$ m; $g = 9{\cdot}8$ m/s^2 2

7. What is resonance? What is sharpness of resonance? 2

8. An electron whose rest mass is $9{\cdot}11\times10^{-31}$ kg is accelerated by potential difference of 50 KV. Calculate the mass of the electron. 2

Group—B

Answer any *four* questions : 4×5

9. (a) What is meant by stable and unstable equilibrium ?

(b) The potential energy of a particle is given by $V(x) = x^4 - 4x^3 - 8x^2 + 48x$. Find the points of stable and unstable equilibrium. 2+3

10. Find the moment of inertia of a uniform rectangular lamina about a diagonal. Also find M.I. for square lamina using the previous result. 4+1

11. Establish the relation among modulus of rigidity, Young modulus and bulk modulus. 5

12. Show how by introducing the idea of reduced mass, a two-body problem under central force can be reduced to a one body problem. 5

13. Show that the resultant of two S.H.M. of the same period but different amplitudes and phases in perpendicular direction is an elliptic motion. For what conditions will the path of the resultant motion be a circle and a straight line ? 3+2

14. Establish relativistic addition of velocities. 5

Group—C

Answer any *one* questions : 1×10

15. (a) Find the speed of a 0·1 Mev electron according to classical and relativistic mechanics? 3

(b) A 200 kg projectile is fired due east with an initial elevation of 30° and initial speed 500 m/s. If the latitude of the place is 60° N, find the magnitude of the total initial coriolis force. 3

(c) A sold sphere rolls down over two different inclined planes of the same height but different inclinations. Will it reach the bottom with the same velocity in each case? Will it take same time? 4

16. (a) Deduce an expression for the couple required to twist a uniform cylinder (wire). What is 'torsional rigidity'.

5+1

(b) Calculate the velocity with which a body must be thrown vertically upward from the surface of the earth, so that it may reach a height 10R, where R is the radius of the earth. 4

$R = 6 \cdot 4 \times 10^6 \text{m}, \ M = 6 \times 10^{24} \text{Kg}$

$G = 6 \cdot 7 \times 10^{-11} \text{N} - \text{m}^2 / \text{kg}^2$

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2018

CBCS

1st Semester

MATHEMATICS

PAPER—GE1T

(Honours)

Full Marks : 60

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Calculus Geometry and Differential Equation

Unit—I

1. Answer any *three* questions : 3×2

(a) If $\lim\limits_{x\to 0} \dfrac{ae^{x}+be^{-x}+2\sin x}{\sin x + x\cos x} = 2$, find the values of a and b.

(Turn Over)

(b) Draw a rough sketch of $y = \cosh x$. 2

(c) Find the nth derivative of $\dfrac{1}{x^2 - a^2}$. 2

(d) Find the range of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave downwards. 2

(e) From any point P on the parabola $y^2 = 4ax$, perpendiculars PM and PN are drawn to the co-ordinate axes. Find the envelope of the line MN. 2

2. Answer any *one* questions : 10×1

(a) i) Trace the curve $xy^2 = a^2(a - x)$ 5

ii) If $y = (\sin^{-1} x)^2$ prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$$ 5

(b) i) Find the asymptotes of the curve

$$y^3 - yx^2 + y^2 + x^2 - 4 = 0$$

ii) Find if there is any point of inflexion on the curve $y - 3 = 6(x - 2)^5$ 5 + 5

Unit—II

3. Answer any *two* of the following : 2×2

(a) Find the differential of arc length for the curve $x = a(1 - \cos\theta)$, $y = a(\theta + \sin\theta)$.

(b) Find the area of the circle $r = 2a\sin\theta$.

(c) Find the reduction formula for $\int \sec^n x\, dx$.

4. Answer any *two* questions : 2×5

(a) Establish the reduction formula for $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$, m, n being positive integers, greater than 1. Hence Calculate $\int_0^{\pi/2} \sin^5 x \cos^6 x\, dx$.

(b) Find the area bounded by the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

(c) Find the volume and surface area generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about initial line.

Unit—III

5. Answer any *three* questions : 3×2

(a) Find the angle through which the axes are to be rotated so that the equation $x\sqrt{3} + y + 6 = 0$ may be reduced to the form $x = c$.

(b) If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ *and* $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, then prove that $pq = -1$.

(c) Find the the equation of the sphere for which the circle

$x^2 + y^2 + z^2 + 2x - 4y + 2z + 5 = 0,$

$x - 2y + 3z + 1 = 0$ is a great circle.

(d) Find the point of intersection of the lines

$r\cos(\theta - \alpha) = p$ and $r\cos(\theta - \beta) = p$

(e) Write down the reflection property of ellipse.

6. Answer any *one* question : 1×5

(a) Show that the distance between two fixed points is unaltered by a rotation of axes.

(b) Find the equation of the cylinder whose generators are parallel to the straight line $2x = y = 3z$ and which passes through the circle $y = 0,\ x^2 + z^2 = 6$.

7. Answer any *one* question : 1×10

(a) i) Prove that the plane $ax + by + cz = 0$ $(a,b,c \neq 0)$ cuts the cone $yz + zx + xy = 0$ in perpendicular straight lines, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

ii) Reduce the equation—

$x^2 + 4xy + 4y^2 + 4x + y - 15 = 0$ to its standard form.

5+5

(b) i) Show that the equation of the circle which passes through the focus of the curve $\frac{l}{r} = 1 - e\cos\theta$ and touches it at the point $\theta = \alpha$ is $r(1 - e\cos\alpha)^2 = l\cos(\theta - \alpha) - el\cos(\theta - 2\alpha)$. 5

ii) Prove that the five normals from a given point to a paraboloid lie on a cone. 5

Unit—IV

8. Answer any *two* questions : 2×2

(a) Determine the order and the degree of the differential equation $\sqrt{y + \left(\frac{dy}{dx}\right)^2} = 1 + x$.

(b) Find an integrating factor of the differential equation $x^2y\,dx - (x^3 + y^3)\,dy = 0$.

(c) Define singular solution of a differential equation.

9. Answer any *one* question : 1×5

(a) Find a solution of the differential equation $\frac{dy}{dx} - y\tan x = 0$ in the form $y = y_1(x)$. Hence solve $\frac{dy}{dx} - y\tan x = \cos x$ by the substitution $y = y_1(x).v(x)$.

(b) By the substitution $x^2 = u$ and $y^2 = v$ reduce the equation $(px - y)(x - py) = 2p$ to clairant's form and find general solution.
