



**JHARGRAM RAJ COLLEGE**  
JHARGRAM – 721 507



**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION – 2021- 2022

SEM: V SUBJECT: MATHEMATICS PAPER: C 12 T (GROUP THEORY - II)

**Date: 05.01.2022**

**Maximum Marks: 10**

**ANSWER ANY FIVE OF THE FOLLOWING**

1. Let  $G$  be a group and  $H$  be subgroup. Set  $X$  to be the set of all left cosets of  $H$ . Prove that  $G$  acts on  $X$  with the following action –

$$g.(aH) = (ga)H$$

2. With the notation as in question no. 1, consider  $G = S_3$  and  $H = \langle (12) \rangle$ . Compute the orbits and stabilizers of all the elements of  $X$ .
3. Let  $G$  be a finite group and  $p$  be a prime divisor of  $O(G)$ . Prove that  $G$  contains an element of prime order  $p$ .
4. Let  $G$  be a finite group having two conjugate classes. Show that  $o(G) = 2$ .
5. Let  $G$  be a non – commutative group of order  $p^3$ , where  $p$  is a prime number. Prove that the centre of the group is a finite subgroup of order  $p$ .
6. Determine the order of  $(10,15,24)$  in  $\mathbb{Z}_{12} \times \mathbb{Z}_{30} \times \mathbb{Z}_{40}$ .

*[the symbols carry their usual meanings]*

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