ANSWER ANY FIVE OF THE FOLLOWING

1. Let G be a group and H be subgroup. Set X to be the set of all left cosets of H. Prove that G acts on X with the following action –

$$g.(aH) = (ga)H$$

2. With the notation as in question no. 1, consider $G = S_3$ and $H = <(12)>$. Compute the orbits and stabilizers of all the elements of X.

3. Let G be a finite group and p be a prime divisor of $O(G)$. Prove that G contains an element of prime order p.

4. Let G be a finite group having two conjugate classes. Show that $o(G) = 2$.

5. Let G be a non – commutative group of order p^3, where p is a prime number. Prove that the centre of the group is a finite subgroup of order p.

6. Determine the order of $(10,15,24)$ in $\mathbb{Z}_{12} \times \mathbb{Z}_{30} \times \mathbb{Z}_{40}$.

[*the symbols carry their usual meanings*]

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