



# JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507



## DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2021- 2022

SEM: III

SUBJECT: MATHEMATICS

PAPER: CC6T

**Full Marks: 10**

**Answer any five questions:**

**(5 × 2 = 10)**

1. Let  $X$  be a non empty set and  $P(X)$  be the power set of  $X$ . Examine if  $P(X)$  is a group under the composition  $*$  defined by  $A * B = A \Delta B = (A - B) \cup (B - A)$ ,  $A, B \in P(X)$ .
2. If each element in a group be its own inverse then prove that the group is abelian.
3. Find all elements of order 5 in the group  $(Z_{20}, +)$ .
4. Prove that centralizer of an element in a group  $G$  is a subgroup of  $G$ .
5. In a group  $G$ ,  $a$  is the only element of order  $n$ , for some  $n \in N$ . Prove that  $a \in Z(G)$ .
6. Find the order of  $(1\ 2\ 3)(5\ 4)$  in  $S_5$ .
7. Let  $S = \{1, x, x^2, \dots, x^5\}$  where  $x = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$ . Prove that  $S$  is a cyclic group under multiplication.
8. Prove that in a cyclic group of even order, there is exactly one element of order 2.