



JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2021- 2022

SEM: III

SUBJECT: MATHEMATICS

PAPER: CC5T

Full Marks: 10

Answer any one Question:

1 × 10 = 10

1. (a) Let $f: [a, b] \rightarrow \mathbb{R}$ be continuous and let the equation $f(x) = 0$ have a finite number of roots in $[a, b]$. Arrange them in the ascending order: $a < x_1 < x_2 < x_3 < \dots < x_n < b$. Prove that in each of the sub intervals $(a, x_1), (x_1, x_2), \dots, (x_n, b)$ the function f retains the same sign.

(b) Let f be a periodic real valued function on real numbers. Show that f is uniformly continuous on \mathbb{R} .

(c) Show that (X, d) is a metric space, where $X = \{\{x_n\}: |x_n| \leq 1, \forall n \in \mathbb{N}\}$, $d(\{x_n\}, \{y_n\}) = \sum_{n=1}^{\infty} \frac{|x_n - y_n|}{2^n}$, for $\{x_n\}, \{y_n\} \in X$.

3 + 3 + 4

2. (a) Prove that in a metric space finite union of closed sets is a closed set.

(b) Prove that a discrete metric space is a Hausdorff space.

(c) Prove that $\sin x^2$ is not uniformly continuous on \mathbb{R} .

4 + 3 + 3