JHARGRAM RAJ COLLEGE

JHARGRAM – 721507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023- 2024

SEM: ISUBJECT: MATHEMATICS PAPER: MATHMJ101(Calculus, Geometry & Differential Equation)

Maximum Marks: 10

Date: 19/12/2023

ANSWER ANY FIVE OF THE FOLLOWING QUESTIONS

1. If $y = \frac{x}{1+x}$, Show that $y_5(0) = 5!$.
2. Find the radius of curvature of $y^2 = 4x$ at the vertex.
3. Obtain a reduction formula for $\int sec^n x\, dx$, hence find $\int sec^6 x dx$.
4. Find the area bounded by the parabolas $x^2 = 4y$ &$y^2 = 4x$.
5. Determine the type of the conic $8x^2 + 10xy + 3y^2 + 22x + 14y + 15 = 0$.
6. Find the polar equation of the tangent to the circle $r = 2d\cos\theta$ at the point whose vectorial angle is θ_1.
7. Find the angle of rotation about the origin which will transform the equation $x^2 - y^2 = 4$into $x'y' + 2 = 0$.
8. Solve: $y(1 + xy)dx + x(1 - xy)dy = 0$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023- 2024

SEM: III SUBJECT: MATHEMATICS PAPER: C5T (Theory of Real Functions & Introduction to Metric Space)

Date: 19/12/2023 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) Show that $\lim_{x\to\infty} \frac{[x]}{x} = 1$.

   (b) If $f: [a, b] \to [a, b]$ is a continuous function, show that $\exists\, c \in [a, b]$ *such that* $f(c) = c$.

   (c) Let (X, d) be an arbitrary metric space then prove that
   $$d^*(x, y) = \frac{d(x,y)}{1+d(x,y)}, \forall\, x, y \in X$$
   is a metric on X.

   $2 + 4 + 4$

2. (a) A function $f: \mathbb{R} \to \mathbb{R}$ is defined by
   $$f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R}\backslash\mathbb{Q} \end{cases}$$
   Prove that f is not continuous at every point of $\mathbb{R}$.

   (b) Let f be continuous on $\mathbb{R}$ and let $f(x) = 0$ when $x \in \mathbb{Q}$. Prove that $f(x) = 0\ \forall\, x \in \mathbb{R}$.

   (c) Find the diameter of the set $\{(x, y): 0 < x < 1, y = e^x\}$ with respect to usual metric on $\mathbb{R}^2$.

   $2 + 4 + 4$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023- 2024

SEM: III SUBJECT: MATHEMATICS PAPER: C 6 T (GROUP THEORY I)

Date: 19/12/2023 **Maximum Marks: 10**

ANSWER ANY FIVE OF THE FOLLOWING

1. If each element in a group be its own inverse, prove that the group is abelian.
2. Find all elements of order 10 in the group $(\mathbb{Z}_{30}, +)$.
3. In a group G, a is the only element of order n for some $n \in \mathbb{N}$. Prove that $a \in Z(G)$.
4. Prove that in a cyclic group of even order there is exactly one element of order 2.
5. In a group G, a is an element of order 30. Find the order of a^{18}.
6. Let G be an abelian group. Prove that the set of all elements of finite order in G forms a subgroup of G.
7. Prove that every cyclic group is abelian.
8. If an abelian group G of order 10 contains an element of order 5. Prove that G must be a cyclic group.

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023

SEM: IIISUBJECT: MATHEMATICS PAPER: C 7 T (NUMERICAL METHODS)

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING QUESTIONS

1) Define the following terms with examples –
   a) Significant digits / figures
   b) Rounding-off error
   c) Significant error
   d) Truncation error
2) If 0.333 is the approximate value of $\frac{1}{3}$, then determine the absolute, relative and percentage error.
3) Three approximate values of the number $\frac{1}{3}$ are given by 0.30, 0.33, and 0.34. which of these three is the best approximation?
4) Find the relative error of the number 8.6 if both of its digits are correct.
5) Explain the shift operator. Establish the following –

   a. $1 + \Delta = E$

   b. $1 - \nabla = E^{-1}$
6) State and prove the fundamental theorem of difference calculus.
7) Define the Newton's Forward difference operator. Prove that Newton's Forward difference operator is a linear operator. Calculate the forward difference table for the function given in the following tabular form –

x	1.00	1.25	1.50	1.75	2.00
e^{-x}	0.3679	0.2865	0.2231	0.1738	0.1353

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023- 2024
SEM: III SUBJECT: MATHEMATICS PAPER: SEC1T (Logic & SET)

Date: 20/12/2023 **Maximum Marks: 5**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) If $|A| = n$ then prove that $|P(A)| = 2^n$.

   (b) A relation ρ is defined on $\mathbb{Z}$ by "$a\rho b\ iff\ 3a + 4b$ is divisible by 7 $\forall\ a, b \in \mathbb{Z}$. Prove that ρ is an equivalence relation on $\mathbb{Z}$.

   3 + 2

2. (a) If $A_n = \left[\frac{1}{n}, 1\right], n \geq 2$ find $\bigcup_{n=2}^{\infty} A_n$ & $\bigcap_{n=2}^{\infty} A_n$

   (b) Show that $(\mathbb{Z}, \leq)$ is not a poset, where $x \leq y$ means "$x\ is\ a\ divisor\ of\ y$" for all $x, y \in \mathbb{Z}$.

   2 + 3

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023- 2024

SEM: V SUBJECT: MATHEMATICS PAPER: C 11 T (PDE & APPLICATIONS)

Date: 19/12/2023 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) Eliminate arbitrary constants from $z = (x-a)^2 + (y-b)^2$ to form the partial differential equation.

   (b) Solve: $px + qy = z,\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

   (c) A particle describes the equiangular spiral $r = ae^{\theta}$ in such a manner the radial acceleration is zero. Prove that the speed and the magnitude of acceleration are proportional to r.

   $2+4+4$

2. (a) Obtain a partial differential equation by eliminating arbitrary constants from the equation $z = ax + (1-a)y + b$.

   (b) Solve: $px - qy = xy,\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

   (c) Prove that in any central orbit the sectorial area traced out by the radius vector to the centre of force increases at a constant rate.

   $2+4+4$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023- 2024

SEM: V SUBJECT: MATHEMATICS PAPER: C 12 T (GROUP THEORY II)

Date: 19/12/2023 **Maximum Marks: 10**

ANSWER ANY FIVE OF THE FOLLOWING

1. Show that $Aut(\mathbb{Z}) \cong \mathbb{Z}_2$.
2. Find the number of inner automorphism of the group S_3.
3. Let G be a group. Show that $Z(G)$ is a characteristic subgroup of G.
4. Let G be a group. Prove that $G^{/} =< [x, y]: x, y \in G =< x^{-1}y^{-1}xy: x, y \in G >$ is a normal subgroup of G.
5. Show that S_3 cannot be written as a direct product of proper subgroup.
6. Can the cyclic group $\mathbb{Z}_{12}$ be expressed as an internal direct product of two proper subgroups?
7. Find all abelian group of order 180.
8. Show that $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_{25} \times \mathbb{Z}_3 \times \mathbb{Z}_2 \cong \mathbb{Z}_{150} \times \mathbb{Z}_6 \times \mathbb{Z}_2$.

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721507

DEPARTMENT OF MATHEMATICS

INTERNAL ASSESSMENT – 2023

SEM: V SUBJECT: MATHEMATICS PAPER: DSE – I (LPP AND GAME THEORY)

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING QUESTIONS

1) Define the following terms–
   a) Decision variables
   b) Legitimate variables
   c) Convex set
   d) Extreme point
   e) Basic feasible solution in connection to a LPP
2) An appliance company manufactures heaters and air conditioners. The production of one heater requires 2 hours in the parts division of the company and 1 hour in the assembly division of the company; the production of one air conditioner requires 1 hour in the parts division of the company and 2 hours in the assembly division of the company. The parts division is operated for at most 8 hours per day and the assembly division is operated for at most 10 hours per day. If the profit realized upon sale is 30 per heater and 50 per air conditioner, then calculate how many heaters and air conditioners should the company manufacture per day to maximize profits by formulating a Linear Programming Problem?
3) Prove that $\left(\frac{22}{5}, \frac{1}{5}, 0\right)$ is a basic feasible solution for the following set of equations –

$$2x_1 + x_2 - 5x_3 = 9$$

$$x_1 + 3x_2 + 7x_3 = 5$$

4) Prove that the set of all convex combinations of a finite number of points is a convex set. Also show that a Hyperplane is a convex set.
5) Put the following L.P.P. in standard form –

$$\text{Minimize } Z = 3x - 4y - z$$

$$\text{Subject to: } x + 3y - 4z \leq 12$$

$$2x - y + z \leq 20$$

$$x - 4y - 5z \geq 5$$

$x \geq 0$, y, z are unrestricted in signs.

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023- 2024

SEM: V SUBJECT: MATHEMATICS PAPER: DSE2 T (PROBABILITY & STATISTICS)

Date: 19/12/2023 **Maximum Marks: 10**

ANSWER ANY FIVE OF THE FOLLOWING

1. An integer is chosen at random from the set $\{1,2,\ldots,10\}$. What is the probability that the integer is divisible by 2 or 3.

2. For n mutually independent events $A_1, A_2, \ldots, A_n$ show that $P(A_1 + A_2 + \cdots + A_n) = 1 - (1 - P(A_1))(1 - P(A_2)) \ldots (1 - P(A_n))$.

3. What is the probability of obtaining a multiple of 3 twice in a throw of 6 dice?

4. Find the constant k such that the function f given by

$$f(x) = \begin{cases} k(x^{[x]} - [x]), & 1 < x < 2 \\ 0, & else \end{cases}$$

is a probability density function.

5. Find the probability density function of distribution whose distribution function is given by

$$f(x) = \begin{cases} \dfrac{x^2}{2}, & 0 < x \le 1 \\ -\dfrac{1}{8} + \dfrac{1}{4}\left(3x - \dfrac{x^2}{2}\right), & 1 < x \le 3 \\ 1, & x > 3 \\ 0, & elsewhere \end{cases}$$

6. Let X be a random variable having probability density function given by $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$. Find the corresponding distribution function.

7. Find expectation for the following density function

$$f(x) = \begin{cases} \dfrac{4x}{5}, & 0 < x \le 1 \\ \dfrac{2}{5}(3 - x), & 1 < x \le 2 \\ 0, & elsewhere \end{cases}$$

8. Find the median of the continuous distribution with probability density function given by

$$f(x) = \begin{cases} \sin x, & 0 \le x \le \dfrac{\pi}{2} \\ 0, & elsewhere \end{cases}$$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023- 2024

SEM: I SUBJECT: MATHEMATICS PAPER: MATSEC01(MATLAB)

Date: 09/04/2024 **Maximum Marks: 20**

GROUP – A

Answer any one question **10 × 1**

1. Write a program to find the sum and maximum of a list of number in an array without library function.
2. Write a program to find the product and minimum of a list of number in a sub-array without library function.
3. Write a program to find the product of a list of number in an array and maximum of a list of number in a sub-array without library function.
4. Write a program to find a sub-matrix of a square matrix.
5. Write a program to find sub-matrix of a rectangular matrix.
6. Write a program to find the column sum and maximum of the given matrix without library function.
7. Write a program to evaluate the column product and minimum of the given matrix without library function.
8. Write a program to evaluate the row product and maximum of the given matrix without library function.
9. Write a program to evaluate the row sum and minimum of the given matrix without library function.
10. Write a program to define the transcendental function $x^{\sin\frac{1}{x^x}}$ over the interval $[\frac{\pi}{2}, \pi]$ with 100 spacing and then find and show the table of its functional values. Plot the function also.
11. Write a program to define the transcendental function $x^{\log\frac{1}{x^x}}$ over the interval [1,5] with 100 spacing and then find and show the table of its functional values. Plot the function also.
12. Write a program to define the transcendental function $x^{e^{\frac{1}{x^x}}}$ over the interval [1,2] with 100 spacing and then find and show the table of its functional values. Plot the function also.
13. Write a program to define the transcendental function $x^{\tan\frac{1}{x^x}}$ over the interval $[\frac{\pi}{4}, \frac{\pi}{2}]$ with 100 spacing and then find and show the table of its functional values. Plot the function also.

a program to define the transcendental function $x^{\sin^{-1}\frac{1}{x^x}}$ over the interval [1,2] with 0 spacing and then find and show the table of its functional values. Plot the function also.

15. Write a program to define the transcendental function $x^{\tan^{-1}\frac{1}{x^x}}$ over the interval [1,2] with 100 spacing and then find and show the table of its functional values. Plot the function also.

GROUP – B

Answer any one question **10×1**

16. Plot the graph of e^{ax+b} and to illustrate the effect of a & b on the graph.
17. Plot the graph of $\log(ax + b)$ and to illustrate the effect of a & b on the graph.
18. Plot the graph of $\sin(ax + b)$ and to illustrate the effect of a & b on the graph.
19. Plot the graph of $\cos(ax + b)$ and to illustrate the effect of a & b on the graph.
20. Plot the graph of $3x^4 - 7x^3 + 12x + 5$, the derivative graph, the second derivative graph and compare them.
21. Plot the graph of $x^5 + 4x^3 + 7x^2 - 8x + 11$, the derivative graph, the second derivative graph and compare them.
22. Sketch the parametric curve Trochoid.
23. Sketch the parametric curve Cycloid.
24. Sketch the parametric curve Epicycloid.
25. Sketch the parametric curve Hypocycloid.
26. Trace the conic $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$ and find the nature of the same.
27. Trace the conic $11x^2 - 4xy + 14y^2 - 58x - 44y + 71 = 0$ and find the nature of the same.
28. Trace the conic $x^2 + 4xy + y^2 - 2x + 2y + 6 = 0$ and find the nature of the same.
29. Trace the conicoid $\frac{x^2}{2} - \frac{y^2}{3} = z$.
30. Trace the conicoid $\frac{x^2}{48} + \frac{y^2}{12} + \frac{z^2}{4} = 1$.
