JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: V SUBJECT: MATHEMATICS PAPER: C 11 T (PDE & APPLICATIONS)

Date: 20/09/2022 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) Eliminate arbitrary constants from $z = (x - a)^2 + (y - b)^2$ to form the partial differential equation.

   (b) Solve: $px + qy = z,\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

   (c) Solve: $y^2(x - y)p + x^2(y - x)q = z(x^2 + y^2),\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

   2 + 4 + 4

2. (a) Obtain a partial differential equation by eliminating a & b from the equation $az + b = a^2x + y$.

   (b) Solve: $px - qy = xy,\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

   (c) Solve: $(y + zx)p - (x + yz)q = x^2 - y^2, p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

   2 + 4 + 4

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023
SEM: V SUBJECT: MATHEMATICS PAPER: C 12 T (GROUP THEORY II)

Date: 20/09/2022 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) Let T be the group of all complex numbers ω such that $|\omega| = 1$. Show that $\mathbb{R}/\mathbb{Z} \cong T$ where $\mathbb{R}$ is the additive group of all real numbers.

   (b) Find all homomorphism from $(\mathbb{Z}_6, +)$ into $(\mathbb{Z}_4, +)$.

   (c) Show that $(\mathbb{Z}_9, +)$ is not a homomorphic image of $(\mathbb{Z}_{16}, +)$.

   $5 + 3 + 2$

2. (a) Prove that $7\mathbb{Z}/56\mathbb{Z} \cong \mathbb{Z}_8$.

   (b) Show that $GL(2, \mathbb{R})/SL(2, \mathbb{R}) \cong \mathbb{R}^*$.

   (c) Prove that up to isomorphism there are only 2 groups of order 4.

   $2 + 3 + 5$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022

SEM: V SUBJECT: MATHEMATICS PAPER: DSE – I (LINEAR PROGRAMMING PROBLEM AND GAME THEORY)

Date: 21/09/2022 **Maximum Marks: 10**

ANSWER ANY THREE OF THE FOLLOWING

1) Define the following –

a. Decision variable

b. Slack variable and surplus variable

c. Objective function

d. Canonical form of a linear programming problem

2) Reduce the following L.P.P. into its standard form –

Maximize $Z = 2x_1 + 3x_2$

Subject to $2x_1 + 3x_2 \leq 300$

$x_1 + x_2 \leq 300$

$x_1 + 3x_2 \geq 240$

Where $x_1, x_2 \geq 0$

3) Define basic feasible solution for a Linear Programming Problem. Determine a basic feasible solution for the following set of constraints connected to an objective function

$$x_1 + 2x_2 - x_3 = 9$$

$$2x_1 - x_2 + x_3 = 5$$

4) Define extreme point of convex set of feasible solutions corresponding to a Linear Programming Problem. Explain the geometrical interpretation of an extreme point.

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022

SEM: V SUBJECT: MATHEMATICS PAPER: DSE – II (PROBABILITY & SATISTICS)

Date: 21/09/2022 **Maximum Marks: 10**

ANSWER ANY THREE OF THE FOLLOWING

1) Define random experiment. Connected to a random experiment let A be an arbitrary event. Prove that the following –

a. $0 \leq P(A) \leq 1$

b. $P(A^c) = 1 - P(A)$

2) A fair coin is tossed twice. Determine the sample space connected to the random experiment as stated. Calculate the probability of the following events –

a. Two heads

b. A head and a tail

3) Two urns containing balls at the following composition. A ball is drawn from the first urn and transferred to the second urn. Finally, a ball is drawn from the second urn. Calculate the probability that the ball drawn from the second urn is white.

Sl. No.	1st Urn	2nd Urn
01.	4 white balls	3 white balls
02.	5 black balls	6 black balls

4) Prove that for a pair of mutually exclusive events A, B connected to a random experiment, $P(A \cup B) = P(A) + P(B)$.

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: III SUBJECT: MATHEMATICS PAPER: C5T (Theory of Real Functions & Introduction to Metric Space)

Maximum Marks: 10

Date: 22/11/2022

ANSWER ANY FOUR OF THE FOLLOWING

1. Show that $\lim_{x\to 0} x\cos\frac{1}{x} = 0$.

2. Show that $\lim_{x\to 0} \sin\frac{1}{x}$ does not exist in $\mathbb{R}$.

3. A function $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x) = x^2, \forall x \in \mathbb{R}$. Prove that f is continuous at every point of $\mathbb{R}$.

4. A function $f:\mathbb{R}\to\mathbb{R}$ is defined by

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R}\setminus\mathbb{Q} \end{cases}$$

Prove that f is not continuous at every point of $\mathbb{R}$.

5. Let f be continuous on $\mathbb{R}$ and let $f(x) = 0$ when $x \in \mathbb{Q}$. Prove that $f(x) = 0 \; \forall \, x \in \mathbb{R}$.

6. Find the point of discontinuity of the function $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x) = [\sin x], \forall \, x \in \mathbb{R}$.

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: III SUBJECT: MATHEMATICS PAPER: C 6 T (GROUP THEORY I)

Date: 22/11/2022 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) Let G be a group and $a, b \in G$, suppose that $a^2 = e$ & $ab^4a = b^7$, prove that $b^{33} = e$.

   (b) Let G be a group. If for all elements a, b, c of $G, ab = ca \Rightarrow b = c$ then show that G is a commutative group.

   (c) If $\beta \in S_7$ & $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$ then find β.

   $5 + 2 + 3$

2. (a) Prove that any finitely generated subgroup of $(\mathbb{Q}, +)$ is cyclic.

   (b) Compute $(3\ 5)(1\ 2\ 4\ 5)(2\ 3\ 1\ 5)$ in S_5.

   (c) Let H be a subgroup of a group G. Show that for any $g \in G, K = gHg^{-1} = \{ghg^{-1}: h \in H\}$ is a subgroup of G and $|K| = |H|$.

   $3 + 2 + 5$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022

SEM: III SUBJECT: MATHEMATICS PAPER: C 7 T 7 (NUMERICAL METHODS)

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING

1) State the necessary and sufficient condition for convergence of the Gauss – Jacobi Iteration method for solving a system of simultaneous linear algebraic equations.
2) State and prove the convergence of the Gauss-Seidel Iterative method.
3) Justify whether Gauss-Seidel Iterative method is applicable to solve the following system of equations or not –

$$x + y + 4z = 9$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

4) Bisection method is also known as the root-bracketing method. Justify the statement.
5) State and prove the condition of convergence for Newton – Raphson method for solving a non – linear or transcendental equation.
6) Determine the positive roots of the equation $x^3 - 3x + 1.06 = 0$, by method of Bisection correct to three places of decimal.
7) For finding the square root of 'a' ($a > 0$) derive the following iteration formula $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$, $n = 0,1,2,3, \ldots$ Where $x_o(> 0)$ is any initial approximation to the actual root and x_n is the n^{th} approximation.
8) Establish the convergence criterion for Regula – Falsi method.
9) Solve the following system of linear equations by Gauss – Seidel iterative method –

$$8x + 2y - 2z = 8$$
$$x - 8y + 3z = -4$$
$$2x + y + 9z = 12$$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022

SEM: III SUBJECT: MATHEMATICS PAPER: SEC – I (LOGIC AND SETS)

Date: 23/11/2022 **Maximum Marks: 10**

ANSWER ANY THREE OF THE FOLLOWING

1) Define the following and explain with examples –

a. Disjunction

b. Conjunction

c. Exclusive disjunction

d. Negation of a proposition

2) Construct the truth table of the following propositions –

i. $p \vee q$

ii. $p \oplus q \wedge r$

3) Verify that $p \wedge q \wedge \sim p$ is a contradiction and $p \rightarrow q \leftrightarrow \sim p \vee q$ is a tautology.

4) Let T denote a tautology (i.e. a statement whose truth value is always true) and F a contradiction. Then, for any statement p, show that –

i. $p \vee T = T$

ii. $p \wedge T = T$

5) Give an example, with justification, of a compound proposition that is neither a tautology nor a contradiction.

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: I SUBJECT: MATHEMATICS PAPER: C1T (Calculus, Geometry & Differential Equation)

Date: 02/12/2022 **Maximum Marks: 10**

ANSWER ANY FIVE OF THE FOLLOWING

1. If $y = \frac{x}{1+x}$, Show that $y_5(0) = 5!$
2. Find the radius of curvature of $y^2 = 4x$ at the vertex.
3. When the axes are turned through an angle, the expression $(ax + by)$ becomes $(a'x' + b'y')$ referred to new axes. Show that $a^2 + b^2 = (a')^2 + (b')^2$.
4. Obtain the equation of the circle lying on the sphere $x^2 + y^2 + z^2 - 2x + 2y - 4z + 3 = 0$ and having its centre at the point $(2,2,-3)$.
5. Show that the equation $(a^2 + b^2)(x^2 + y^2) = (ax + by - ab)^2$ represents a parabola of latus rectum $\frac{2ab}{\sqrt{a^2+b^2}}$.
6. Determine the values of a & b such that $\lim_{x\to 0} \frac{x(1+a\cos x)-b\sin x}{x^3} = 1$.
7. Solve: $\frac{dy}{dx} + \frac{y}{x}\ln y = \frac{y}{x^2}(\ln y)^2$
8. Solve: $xdx + ydy + \frac{xdy - ydx}{x^2+y^2} = 0$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022
SEM: I SUBJECT: MATHEMATICS PAPER: C – II (ALGEBRA)

Maximum Marks: 10

Date: 02.12.2022

ANSWER ANY FIVE OF THE FOLLOWING QUESTIONS

1. Find the least positive value of the expression $567x + 315y + 30z, x, y, z \in \mathbb{Z}$.
2. Let $V(R)$ be the vector space of all polynomial functions in x with real coefficients. The linear operators D, T on $V(R)$ are defined as –
   $D[f(x)] = \frac{df(x)}{dx}$ and $T[f(x)] = \int_0^x f(x)dx$ for all $f(x) \in V(R)$. Show that $DT = I$.
   Is D one – one?
3. If $(\alpha, \beta, \gamma), (\delta, \varepsilon)$ be the ordered bases of the vector spaces U, V respectively and $T: U \to V$ be a linear transformation such that $T(\alpha) = \delta - 2\varepsilon, T(\beta) = \gamma + 3\varepsilon, T(\gamma) = 2\delta + \varepsilon$. Determine the matrix representation of the transformation T relative to the bases $(\alpha, \beta, \beta + \gamma)$ of U and $(\delta - \varepsilon, \varepsilon)$ of V.
4. Prove that $\arg(z) - \arg(-z) = \pm\pi$ *according as* $\arg(z) > 0$ *and* $\arg(-z) < 0$.
5. Prove that $Re(z) + Im(z) \leq \sqrt{2}z$
6. Prove by the method of induction $2.7^n + 3.5^n - 5$ is divisible by 24.
7. Compute the matrix $E_{23}E_1(2)E_{12}(-2)E_{21}$ for elementary matrix of order 4.
8. Reduce the following matrix to row reduced echelon form –

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

Jhargram Raj College Department of Mathematics ★ Govt. of W.B. ★

INTERNAL EXAMINATION – 2022- 2023

SEM: IV SUBJECT: MATHEMATICS PAPER: C 13 T (METRIC SPACE AND COMPLEX ANALYSIS)

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING

1. Let (X, d) be a metric space. Define $d^*(x,y) = \frac{d(x,y)}{1+d(x,y)}$, for all $x, y \in X$. Prove that $d^*$is a bounded metric on X.
2. Show that the following conditions
   a. $d(x,y) = 0$ iff $x = y$ $(x, y \in X)$
   b. $d(x,z) \leq d(x,y) + d(y,z). \forall x, y, z \in X$

   are not sufficient to ensure that the mapping $d: X \times X \to \mathbb{R}$ is a metric on the set X.
3. Prove that a subset A of a metric space (X, d) is open iff $A \cap db(A) = \varphi$.
4. Show that a real valued function f on a metric space (X, d) is continuous if and only if for each $a \in \mathbb{R}, \{x \in X: f(x) > a\}$ and $\{x \in X: f(x) < a\}$ are open sets in X.
5. Show that a surjection $f: [a, b] \to B (B \subseteq \mathbb{R})$, where B is not closed in $\mathbb{R}$ cannot be continuous.
6. Let u and v denote the real and imaginary components of the function f defined by means of the equations

$$f(z) = \begin{cases} \frac{\bar{z}^2}{z}, \text{ when } z \neq 0 \\ 0, \text{ when } z = 0 \end{cases}$$

   Verify that the Cauchy – Riemann equations are satisfied at the origin $z = (0,0)$.
7. Let a function f be analytic everywhere in a domain D. Prove that if $f(z)$ is real - valued for all $z \in D$ then $f(z)$ must be constant throughout D.
8. Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ when $u(x, y) = 2x(1 - y)$.

****

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – VI, INTERNAL ASSESSMENT, 2022-23

Sub: MATHEMATICS, Course – C14T

Marks: 10

ver any five questions:

Time: 30 m.

$(2 \times 5 = 10)$

1. Find all the ideals of the ring $\mathbb{Z}[x]/(2, x^3 + 1)$.
2. Fin all irreducible polynomials of degree 3 in $\mathbb{Z}_2(x)$.
3. Examine if the polynomial $x^2 + 4x - 2$ is irreducible over $\mathbb{Q}[x]$.
4. Let $V = C([0,1])$ and define $\langle f, g\rangle = \int_0^{\frac{1}{2}} f(t)g(t)dt$. Is this an inner product on V?
5. Apply the Gram-Schmidt process to the vectors $(1,0,1), (1,0,-1), (0,3,4)$ to obtain an orthonormal basis for $\mathbb{R}^3$ wi the standard inner product.
5. Let $V = \mathbb{R}^2$ and $T: \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $T(x) = (2x - 2y, -2x + 5y)$,then show that T is self-adjoint operator.
7. Show that the matrix $\begin{pmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix}$ is diagonalisable.
8. Let $\beta = \{(2,1), (3,1)\}$ be an ordered basis for $\mathbb{R}^2$. Find the dual basis of β.

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: VI SUBJECT: MATHEMATICS PAPER: DSE3T (Number Theory)

Date: 30/05/2023 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) For any integer $n > 1$ prove that $\tau(n) \leq 2\sqrt{n}$, where $\tau(n)$ is number of positive divisor of n.

   (b) Prove that if $n \geq 1$ then $\frac{(2n)!}{(n!)^2}$ is an even integer.

   (c) Prove that if $\gcd(a, n) = \gcd(a-1, n) = 1$, then
   $$1 + a + a^2 + \cdots + a^{\varphi(n)-1} \equiv 0 (mod\ n).$$

   $3 + 3 + 4$

2. (a) Prove that for $n > 1$, the sum of the positive integers less than n and relatively prime to n is $\frac{n\varphi(n)}{2}$.

   (b) Verify that $1000!$ terminates in 249 zeros.

   (c) If f is a multiplicative function then prove that $\sum_{d|n} f(d)$ is also a multiplicative function.

   $3 + 2 + 5$

****

JHARGRAM RAJ COLLEGE
JHARGRAM – 721 507
DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: VI SUBJECT: MATHEMATICS PAPER: DSE – 4 (MATHEMATICAL MODELLING)

Maximum Marks: 10

ANSWER THE FOLLOWING QUESTIONS

1. Show that $L(e^{at}t^{\alpha}) = \frac{\Gamma(\alpha+1)}{(s-a)^{\alpha+1}}$.

2. An object having mass m is initially at rest and receives a blow, or impulse, of strength P at $t = 0, t = a, t = 2a$ etc. The equation may be written

$$m\frac{dv}{dt} = P[\delta(t) + \delta(t-a) + \delta(t-2a) + \cdots \ldots\ldots\ldots\ldots\ldots\ldots.]$$

Find the velocity as a function of time.

3. Solve the following differential equation –

$x''(t) + 4x(t) = e^t$ subject to the initial conditions $x(0) = 0$ and $x'(0) = 1$ by application of Laplace transform.

4. Solve the following differential equation by power series solution –

$\frac{d^2y}{dx^2} + y = 0$.

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023
ACADEMIC YEAR : 2022 - 2023

SEM: II | SUBJECT: MATHEMATICS | PAPER: C3T

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING

1. If $a \epsilon R$ and $0 \leq a < \frac{1}{n}$ for every natural number n, prove that $a = 0$.

2. Find $Sup\ A$ and $Inf\ A$, Where $A = \left\{\frac{n+(-1)^n}{n} ; n \epsilon N\right\}$.

3. Prove that the set $S = \{x \epsilon R: Sinx \neq 0\}$ is an open set.

4. Show that a finite set has no limit point.

5. Prove that the set $\left\{\frac{1}{m} + \frac{1}{n} : m\epsilon N, n\epsilon N\right\}$ is closed in R.

6. Show that the $\{x_n\}$sequence is a Null sequence, where $x_n = \frac{n!}{n^n}$

7. Use Sandwich Theorem to prove that
$$lim\left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2}\right] = 0.$$

****

Jhargram Raj College Department of Mathematics Govt. of W...

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: II SUBJECT: MATHEMATICS PAPER: C 4 T (Differential Equation & Vector Calculus)

Date: 25/07/2023 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) Determine the 1st three approximated solutions of the following Initial Value Problem (IVP) –

$$\frac{dy}{dx} = 2y, y(0) = 1$$

(b) Show that the following differential equation –

$$(D^4 - D^3 - 3D^2 + 5D - 2)y = 0, D = \frac{d}{dx}$$

has only two linearly independent solutions of the form $y = e^{mx}$. But verify that $y = e^x, y = xe^x, y = x^2e^x, y = e^{-2x}$ are four linearly independent solutions of the given equation. Hence write down the complete solution.

(c) If $\vec{a}$ & $\vec{b}$ be two non collinear vectors such that $\vec{a} = \vec{c} + \vec{d}$ where $\vec{c}$ is a vector parallel to $\vec{b}$ & $\vec{d}$ is a vector perpendicular to $\vec{b}$, then obtain expressions for $\vec{c}$ & $\vec{d}$ in terms of $\vec{a}$ & $\vec{b}$.

$4 + 3 + 3$

2. (a) Solve the following differential equation by the method of variation of parameters –

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = \log x \ (x > 0)$$

it being given that $y = x$ and $y = {}^1/_x$ are two linearly independent solutions of its reduced equation.

(b) Prove that $\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}\right] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

(c) Solve the following differential equation –

$$\frac{dx}{y^2 + yz + z^2} = \frac{dy}{z^2 + zx + x^2} = \frac{dz}{x^2 + xy + y^2}$$

$3 + 2 + 5$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2023
ACADEMIC YEAR: 2022 - 2023
SEM: IV
SUBJECT: MATHEMATICS
PAPER: C8T

Maximum Marks: 10

ANSWER ANY FOUR OF THE FOLLOWING

1. A function f is defined by $[0,1]$ by
$$f(0) = 0$$
$$f(x) = (-1)^{r-1}, \frac{1}{r+1} < x \leq \frac{1}{r}, \text{ for } r = 1,2,3 \dots$$
Show that f is integrable on $[0,1]$

2. Let $f(x) = [x], x\epsilon[0,3]$.Evaluate $\int_0^3 f$.

3. A function f is defined by $[0,1]$ by
$$f(x) = sinx, x \text{ is rational.}$$
$$= x, \quad x \text{ is irrational.}$$
Show that f is not integrable on $\left[0, \frac{\pi}{2}\right]$

4. Let $f_n(x) = x^{n-1} - x^n, x\epsilon[0,1]$. Prove that the sequence $\{f_n\}$ is uniformly convergent on $[0,1]$.

5. Let $f_n(x) = \frac{nx}{1+nx}, x\epsilon[0,1]$. Show that the sequence $\{f_n\}$ is not uniformly convergent on $[0,1]$.

6. Let $f_n(x) = x^2e^{-nx}, x\epsilon[0,\infty]$. Show that the sequence $\{f_n\}$ is uniformly convergent on $[0,\infty]$.

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: IV SUBJECT: MATHEMATICS PAPER: C 9 T (Multivariable Calculus)

Date: 24/07/2023 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. (a) Let $f(x) = \begin{cases} \frac{x^2y^2}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}$

   Show that $f_{xy}(0,0) = f_{yx}(0,0)$ although f does not satisfy conditions of Schwarz's theorem.

   (b) Check that whether $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x-y}$ exists or not.

   (c) Change the order of integration & hence evaluate

   $\int_2^4 \int_{\frac{4}{x}}^{\frac{20-4x}{8-x}} (4-y)\,dy\,dx$

   $5+2+3$

2. (a) Show that $f(x,y) = \sqrt{|xy|}$ is not differentiable at $(0,0)$.

   (b) Compute $\iint \frac{2x^2+y^2}{xy}\,dx\,dy$ taken over the area in the positive quadrant of the xy plane bounded by the curves $x^2+y^2=h^2, x^2+y^2=k^2, y^2=4ax, y^2=4bx, a,b>0$

   (c) Evaluate $\iiint z^2\,dx\,dy\,dz$ over the region E common to the surfaces $x^2+y^2+z^2=a^2$ & $x^2+y^2=ax$.

   $3+2+5$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: IV SUBJECT: MATHEMATICS PAPER: C 10 T (RING THEORY AND LINEAR ALGEBRA - I)

Date: 25/07/2023 **Maximum Marks: 10**

ANSWER ANY ONE OF THE FOLLOWING

1. If $(R, +, .)$ Be a ring such that $(R, +)$ is a cyclic group, prove that the ring is a commutative ring. Also, deduce that a ring of prime number of elements is always commutative ring.

2. Let $(R, +, .)$ Be a ring with unity element I and $a \in R$. If there exists a unique element $b \in R$ such that $ab = I$, prove that $ba = I$ too and a is a unit.

3. Let D is an integral domain in which the identity element is the only element which is its own inverse. Prove that the characteristic of D is 2.

4. Let a be a fixed element in a ring $(R, +, .)$ and let $C(a) = \{x \in R: xa = ax\}$. Prove that $C(a)$ is a subring of $(R, +, .)$.

5. Let $\alpha_1, \alpha_2, \alpha_3$ are vectors in real vector space V such that $\alpha_1 + \alpha_2 + \alpha_3 = 0$. Prove that $L(\alpha_1, \alpha_2) = L(\alpha_2, \alpha_3) = L(\alpha_1, \alpha_3)$.

6. Let V be a real vector space with a basis $\{\alpha_1, \alpha_2, \ldots \ldots, \alpha_n\}$. Examine if $\{\alpha_1 + \alpha_2, \alpha_2 + \alpha_3, \ldots \ldots, \alpha_n + \alpha_1\}$ is also a basis of V.

7. Determine the linear mapping $T: R^3 \to R^3$ that maps the basis vectors $(0,1,1), (1,0,1), (1,1,0)$ of R^3 to the vectors $(2,1,1), (1,2,1), (1,1,2)$ respectively. Find also the kernel and image of the transformation.

8. Let T be a linear operator on a vector space V over a field F. Prove that –
   - $Ker\ (T) \subset Ker(T^2)$
   - $Im\ (T^2) \subset Im\ (T)$

****

JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023

SEM: IV SUBJECT: MATHEMATICS PAPER: SEC 2 (GRAPH THEORY)

Date: 25/07/2023

Maximum Marks: 5

ANSWER ANY THREE OF THE FOLLOWING

1. Show that a complete graph with n vertices consists of $\frac{n(n-1)}{2}$ edges.

2. Let G be a simple graph with at most $2n$ vertices. If the degree of each vertex is at least n, then show that the graph is connected.

3. Let G be a graph and u, v be two vertices of G such that $u \neq v$. If there is a trail from u to v, then show that there is a path from u to v.

4. Prove that a graph has a circuit if the degree of each vertex is an even positive integer.

****