

**JHARGRAM RAJ COLLEGE**  
JHARGRAM - 721 507



**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION - 2022- 2023  
SEM: V SUBJECT: MATHEMATICS PAPER: C 11 T (PDE & APPLICATIONS)

Date: 20/09/2022

Maximum Marks: 10

**ANSWER ANY ONE OF THE FOLLOWING**

1. (a) Eliminate arbitrary constants from  $z = (x - a)^2 + (y - b)^2$  to form the partial differential equation.

(b) Solve:  $px + qy = z, p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ .

(c) Solve:  $y^2(x - y)p + x^2(y - x)q = z(x^2 + y^2), p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ .

2 + 4 + 4

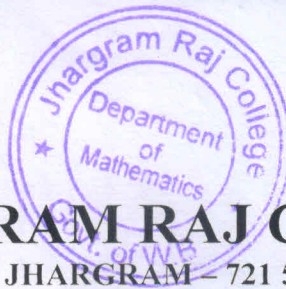
2. (a) Obtain a partial differential equation by eliminating  $a$  &  $b$  from the equation  $az + b = a^2x + y$ .

(b) Solve:  $px - qy = xy, p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ .

(c) Solve:  $(y + zx)p - (x + yz)q = x^2 - y^2, p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

2 + 4 + 4

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# JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507



## DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023  
SEM: V SUBJECT: MATHEMATICS PAPER: C 12 T (GROUP THEORY II)

Date: 20/09/2022

Maximum Marks: 10

### ANSWER ANY ONE OF THE FOLLOWING

- (a) Let  $T$  be the group of all complex numbers  $\omega$  such that  $|\omega| = 1$ . Show that  $\mathbb{R}/\mathbb{Z} \cong T$  where  $\mathbb{R}$  is the additive group of all real numbers.  
(b) Find all homomorphism from  $(\mathbb{Z}_6, +)$  into  $(\mathbb{Z}_4, +)$ .  
(c) Show that  $(\mathbb{Z}_9, +)$  is not a homomorphic image of  $(\mathbb{Z}_{16}, +)$ .

5 + 3 + 2

- (a) Prove that  $7\mathbb{Z}/56\mathbb{Z} \cong \mathbb{Z}_8$ .  
(b) Show that  $GL(2, \mathbb{R})/SL(2, \mathbb{R}) \cong \mathbb{R}^*$ .  
(c) Prove that up to isomorphism there are only 2 groups of order 4.

2 + 3 + 5

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**JHARGRAM RAJ COLLEGE**  
JHARGRAM - 721507

**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION - 2022  
SEM: V SUBJECT: MATHEMATICS PAPER: DSE - I (LINEAR PROGRAMMING PROBLEM AND  
GAME THEORY)

Date: 21/09/2022

Maximum Marks: 10

**ANSWER ANY THREE OF THE FOLLOWING**

- 1) Define the following –
  - a. Decision variable
  - b. Slack variable and surplus variable
  - c. Objective function
  - d. Canonical form of a linear programming problem

- 2) Reduce the following L.P.P. into its standard form –

Maximize  $Z = 2x_1 + 3x_2$   
Subject to  $2x_1 + 3x_2 \leq 300$   
 $x_1 + x_2 \leq 300$   
 $x_1 + 3x_2 \geq 240$

Where  $x_1, x_2 \geq 0$

- 3) Define basic feasible solution for a Linear Programming Problem. Determine a basic feasible solution for the following set of constraints connected to an objective function

$$x_1 + 2x_2 - x_3 = 9$$
$$2x_1 - x_2 + x_3 = 5$$

- 4) Define extreme point of convex set of feasible solutions corresponding to a Linear Programming Problem. Explain the geometrical interpretation of an extreme point.

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**JHARGRAM RAJ COLLEGE**  
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**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION – 2022  
SEM: V SUBJECT: MATHEMATICS PAPER: DSE – II (PROBABILITY & STATISTICS)

Date: 21/09/2022

Maximum Marks: 10

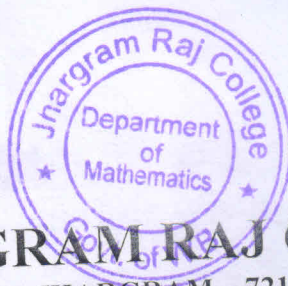
**ANSWER ANY THREE OF THE FOLLOWING**

- 1) Define random experiment. Connected to a random experiment let  $A$  be an arbitrary event. Prove that the following –
  - a.  $0 \leq P(A) \leq 1$
  - b.  $P(A^c) = 1 - P(A)$
- 2) A fair coin is tossed twice. Determine the sample space connected to the random experiment as stated. Calculate the probability of the following events –
  - a. Two heads
  - b. A head and a tail
- 3) Two urns containing balls at the following composition. A ball is drawn from the first urn and transferred to the second urn. Finally, a ball is drawn from the second urn. Calculate the probability that the ball drawn from the second urn is white.

Sl. No.	1 <sup>st</sup> Urn	2 <sup>nd</sup> Urn
01.	4 white balls	3 white balls
02.	5 black balls	6 black balls

- 4) Prove that for a pair of mutually exclusive events  $A, B$  connected to a random experiment,  $P(A \cup B) = P(A) + P(B)$ .

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**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION - 2022- 2023  
SEM: III SUBJECT: MATHEMATICS PAPER: C5T (Theory of Real Functions & Introduction to Metric Space)

Date: 22/11/2022

Maximum Marks: 10

ANSWER ANY FOUR OF THE FOLLOWING

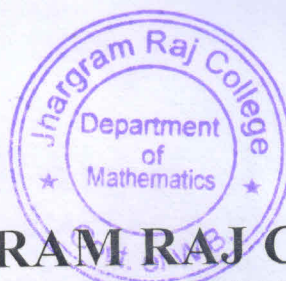
1. Show that  $\lim_{x \rightarrow 0} x \cos \frac{1}{x} = 0$ .
2. Show that  $\lim_{x \rightarrow 0} \sin \frac{1}{x}$  does not exist in  $\mathbb{R}$ .
3. A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x^2, \forall x \in \mathbb{R}$ . Prove that  $f$  is continuous at every point of  $\mathbb{R}$ .
4. A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

Prove that  $f$  is not continuous at every point of  $\mathbb{R}$ .

5. Let  $f$  be continuous on  $\mathbb{R}$  and let  $f(x) = 0$  when  $x \in \mathbb{Q}$ . Prove that  $f(x) = 0 \forall x \in \mathbb{R}$ .
6. Find the point of discontinuity of the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = [\sin x], \forall x \in \mathbb{R}$ .

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**JHARGRAM RAJ COLLEGE**  
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**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION – 2022- 2023  
SEM: III SUBJECT: MATHEMATICS PAPER: C 6 T (GROUP THEORY I)

Date: 22/11/2022

Maximum Marks: 10

**ANSWER ANY ONE OF THE FOLLOWING**

1. (a) Let  $G$  be a group and  $a, b \in G$ , suppose that  $a^2 = e$  &  $ab^4a = b^7$ ,  
prove that  $b^{33} = e$ .

(b) Let  $G$  be a group. If for all elements  $a, b, c$  of  $G$ ,  $ab = ca \Rightarrow b = c$   
then show that  $G$  is a commutative group.

(c) If  $\beta \in S_7$  &  $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$  then find  $\beta$ .

5 + 2 + 3

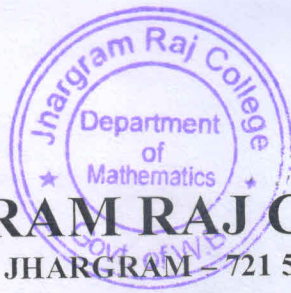
2. (a) Prove that any finitely generated subgroup of  $(\mathbb{Q}, +)$  is cyclic.

(b) Compute  $(3\ 5)(1\ 2\ 4\ 5)(2\ 3\ 1\ 5)$  in  $S_5$ .

(c) Let  $H$  be a subgroup of a group  $G$ . Show that for any  $g \in G$ ,  $K =$   
 $gHg^{-1} = \{ghg^{-1} : h \in H\}$  is a subgroup of  $G$  and  $|K| = |H|$ .

3 + 2 + 5

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# JHARGRAM RAJ COLLEGE

JHARGRAM - 721 507



## DEPARTMENT OF MATHEMATICS

SEM: III

INTERNAL EXAMINATION - 2022  
SUBJECT: MATHEMATICS PAPER: C 7 T 7 (NUMERICAL METHODS)

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING

- 1) State the necessary and sufficient condition for convergence of the Gauss - Jacobi Iteration method for solving a system of simultaneous linear algebraic equations.
- 2) State and prove the convergence of the Gauss-Seidel Iterative method.
- 3) Justify whether Gauss-Seidel Iterative method is applicable to solve the following system of equations or not -

$$x + y + 4z = 9$$

$$8x - 3y + 2z = 20$$

$$4x + 11y - z = 33$$

- 4) Bisection method is also known as the root-bracketing method. Justify the statement.
- 5) State and prove the condition of convergence for Newton - Raphson method for solving a non - linear or transcendental equation.
- 6) Determine the positive roots of the equation  $x^3 - 3x + 1.06 = 0$ , by method of Bisection correct to three places of decimal.
- 7) For finding the square root of 'a' ( $a > 0$ ) derive the following iteration formula  $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ ,  $n = 0, 1, 2, 3, \dots$  Where  $x_0 (> 0)$  is any initial approximation to the actual root and  $x_n$  is the  $n^{\text{th}}$  approximation.
- 8) Establish the convergence criterion for Regula - Falsi method.
- 9) Solve the following system of linear equations by Gauss - Seidel iterative method -

$$8x + 2y - 2z = 8$$

$$x - 8y + 3z = -4$$

$$2x + y + 9z = 12$$

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**JHARGRAM RAJ COLLEGE**  
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**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION – 2022  
SEM: III SUBJECT: MATHEMATICS PAPER: SEC – I (LOGIC AND SETS)

Date: 23/11/2022

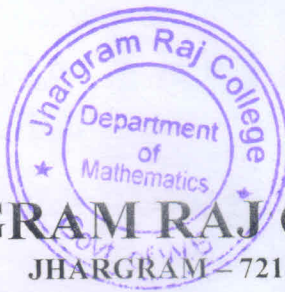
Maximum Marks: 10

**ANSWER ANY THREE OF THE FOLLOWING**

- 1) Define the following and explain with examples –
  - a. Disjunction
  - b. Conjunction
  - c. Exclusive disjunction
  - d. Negation of a proposition
- 2) Construct the truth table of the following propositions –
  - i.  $p \vee q$
  - ii.  $p \oplus q \wedge r$
- 3) Verify that  $p \wedge q \wedge \sim p$  is a contradiction and  $p \rightarrow q \leftrightarrow \sim p \vee q$  is a tautology.
- 4) Let  $T$  denote a tautology (i.e. a statement whose truth value is always true) and  $F$  a contradiction. Then, for any statement  $p$ , show that –
  - i.  $p \vee T = T$
  - ii.  $p \wedge T = \phi$   ~~$\phi$~~   $\alpha$
- 5) Give an example, with justification, of a compound proposition that is neither a tautology nor a contradiction.

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**JHARGRAM RAJ COLLEGE**  
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**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION - 2022- 2023

SEM: I SUBJECT: MATHEMATICS PAPER: CIT (Calculus, Geometry & Differential Equation)

Date: 02/12/2022

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING

1. If  $y = \frac{x}{1+x}$ , Show that  $y_5(0) = 5!$
2. Find the radius of curvature of  $y^2 = 4x$  at the vertex.
3. When the axes are turned through an angle, the expression  $(ax + by)$  becomes  $(a'x' + b'y')$  referred to new axes. Show that  $a^2 + b^2 = (a')^2 + (b')^2$ .
4. Obtain the equation of the circle lying on the sphere  $x^2 + y^2 + z^2 - 2x + 2y - 4z + 3 = 0$  and having its centre at the point  $(2, 2, -3)$ .
5. Show that the equation  $(a^2 + b^2)(x^2 + y^2) = (ax + by - ab)^2$  represents a parabola of latus rectum  $\frac{2ab}{\sqrt{a^2 + b^2}}$ .
6. Determine the values of  $a$  &  $b$  such that  $\lim_{x \rightarrow 0} \frac{x(1+a \cos x) - b \sin x}{x^3} = 1$ .
7. Solve:  $\frac{dy}{dx} + \frac{y}{x} \ln y = \frac{y}{x^2} (\ln y)^2$
8. Solve:  $x dx + y dy + \frac{x dy - y dx}{x^2 + y^2} = 0$

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**JHARGRAM RAJ COLLEGE**  
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**DEPARTMENT OF MATHEMATICS**

INTERNAL EXAMINATION - 2022  
SEM: I SUBJECT: MATHEMATICS PAPER: C - II (ALGEBRA)

Date: 02.12.2022

Maximum Marks: 10

**ANSWER ANY FIVE OF THE FOLLOWING QUESTIONS**

1. Find the least positive value of the expression  $567x + 315y + 30z$ ,  $x, y, z \in \mathbb{Z}$ .
2. Let  $V(R)$  be the vector space of all polynomial functions in  $x$  with real coefficients. The linear operators  $D, T$  on  $V(R)$  are defined as -  
 $D[f(x)] = \frac{df(x)}{dx}$  and  $T[f(x)] = \int_0^x f(x)dx$  for all  $f(x) \in V(R)$ . Show that  $DT = I$ .  
Is  $D$  one - one?
3. If  $(\alpha, \beta, \gamma), (\delta, \varepsilon)$  be the ordered bases of the vector spaces  $U, V$  respectively and  $T: U \rightarrow V$  be a linear transformation such that  $T(\alpha) = \delta - 2\varepsilon, T(\beta) = \gamma + 3\varepsilon, T(\gamma) = 2\delta + \varepsilon$ . Determine the matrix representation of the transformation  $T$  relative to the bases  $(\alpha, \beta, \beta + \gamma)$  of  $U$  and  $(\delta - \varepsilon, \varepsilon)$  of  $V$ .
4. Prove that  $\arg(z) - \arg(-z) = \pm\pi$  according as  $\arg(z) > 0$  and  $\arg(-z) < 0$ .
5. Prove that  $Re(z) + Im(z) \leq \sqrt{2}z$
6. Prove by the method of induction  $2 \cdot 7^n + 3 \cdot 5^n - 5$  is divisible by 24.
7. Compute the matrix  $E_{23}E_1(2)E_{12}(-2)E_{21}$  for elementary matrix of order 4.
8. Reduce the following matrix to row reduced echelon form -

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

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# JHARGRAM RAJ COLLEGE

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## DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION - 2022-2023

SUBJECT: MATHEMATICS PAPER: C 13 T (METRIC SPACE AND COMPLEX ANALYSIS)

Maximum Marks: 10

### ANSWER ANY FIVE OF THE FOLLOWING

1. Let  $(X, d)$  be a metric space. Define  $d^*(x, y) = \frac{d(x, y)}{1+d(x, y)}$ , for all  $x, y \in X$ . Prove that  $d^*$  is a bounded metric on  $X$ .
2. Show that the following conditions
  - a.  $d(x, y) = 0$  iff  $x = y$  ( $x, y \in X$ )
  - b.  $d(x, z) \leq d(x, y) + d(y, z)$ .  $\forall x, y, z \in X$

are not sufficient to ensure that the mapping  $d: X \times X \rightarrow \mathbb{R}$  is a metric on the set  $X$ .

3. Prove that a subset  $A$  of a metric space  $(X, d)$  is open iff  $A \cap db(A) = \varnothing$ .
4. Show that a real valued function  $f$  on a metric space  $(X, d)$  is continuous if and only if for each  $a \in \mathbb{R}$ ,  $\{x \in X: f(x) > a\}$  and  $\{x \in X: f(x) < a\}$  are open sets in  $X$ .
5. Show that a surjection  $f: [a, b] \rightarrow B$  ( $B \subseteq \mathbb{R}$ ), where  $B$  is not closed in  $\mathbb{R}$  cannot be continuous.
6. Let  $u$  and  $v$  denote the real and imaginary components of the function  $f$  defined by means of the equations

$$f(z) = \begin{cases} \bar{z}^2, & \text{when } z \neq 0 \\ 0, & \text{when } z = 0 \end{cases}$$

Verify that the Cauchy - Riemann equations are satisfied at the origin  $z = (0, 0)$ .

7. Let a function  $f$  be analytic everywhere in a domain  $D$ . Prove that if  $f(z)$  is real-valued for all  $z \in D$  then  $f(z)$  must be constant throughout  $D$ .
8. Show that  $u(x, y)$  is harmonic in some domain and find a harmonic conjugate  $v(x, y)$  when  $u(x, y) = 2x(1 - y)$ .

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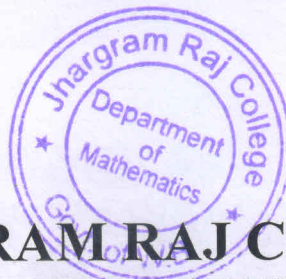
DEPT. OF MATHEMATICS  
JHARGRAM RAJ COLLEGE  
B.Sc(H) Sem – VI, INTERNAL ASSESSMENT, 2022-23  
Sub: MATHEMATICS, Course – C14T

Time: 30 m.  
(2 × 5 = 10)

Marks: 10

Answer any five questions:

1. Find all the ideals of the ring  $\mathbb{Z}[x]/(2, x^3 + 1)$ .
2. Find all irreducible polynomials of degree 3 in  $\mathbb{Z}_2[x]$ .
3. Examine if the polynomial  $x^2 + 4x - 2$  is irreducible over  $\mathbb{Q}[x]$ .
4. Let  $V = C([0,1])$  and define  $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$ . Is this an inner product on  $V$ ?
5. Apply the Gram-Schmidt process to the vectors  $(1,0,1), (1,0,-1), (0,3,4)$  to obtain an orthonormal basis for  $\mathbb{R}^3$  with the standard inner product.
5. Let  $V = \mathbb{R}^2$  and  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is defined by  $T(x) = (2x - 2y, -2x + 5y)$ , then show that  $T$  is self-adjoint operator.
7. Show that the matrix  $\begin{pmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix}$  is diagonalisable.
8. Let  $\beta = \{(2,1), (3,1)\}$  be an ordered basis for  $\mathbb{R}^2$ . Find the dual basis of  $\beta$ .



# JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507



## DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023  
SEM: VI SUBJECT: MATHEMATICS PAPER: DSE3T (Number Theory)

Date: 30/05/2023

Maximum Marks: 10

### ANSWER ANY ONE OF THE FOLLOWING

1. (a) For any integer  $n > 1$  prove that  $\tau(n) \leq 2\sqrt{n}$ , where  $\tau(n)$  is number of positive divisor of  $n$ .

(b) Prove that if  $n \geq 1$  then  $\frac{(2n)!}{(n!)^2}$  is an even integer.

(c) Prove that if  $\gcd(a, n) = \gcd(a - 1, n) = 1$ , then  
 $1 + a + a^2 + \dots + a^{\varphi(n)-1} \equiv 0 \pmod{n}$ .

3 + 3 + 4

2. (a) Prove that for  $n > 1$ , the sum of the positive integers less than  $n$  and relatively prime to  $n$  is  $\frac{n\varphi(n)}{2}$ .

(b) Verify that  $1000!$  terminates in 249 zeros.

(c) If  $f$  is a multiplicative function then prove that  $\sum_{d|n} f(d)$  is also a multiplicative function.

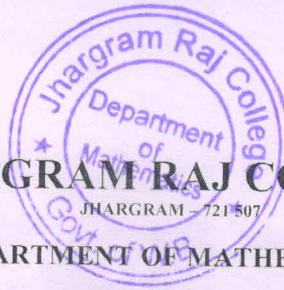
3 + 2 + 5

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# JHARGRAM RAJ COLLEGE

DEPARTMENT OF MATHEMATICS



INTERNAL EXAMINATION – 2022- 2023  
SEM: VI SUBJECT: MATHEMATICS PAPER: DSE – 4 (MATHEMATICAL MODELLING)

Maximum Marks: 10

### ANSWER THE FOLLOWING QUESTIONS

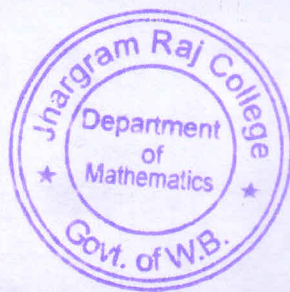
1. Show that  $L(e^{at}t^\alpha) = \frac{\Gamma(\alpha+1)}{(s-a)^{\alpha+1}}$ .
2. An object having mass  $m$  is initially at rest and receives a blow, or impulse, of strength  $P$  at  $t = 0, t = a, t = 2a$  etc. The equation may be written

$$m \frac{dv}{dt} = P[\delta(t) + \delta(t - a) + \delta(t - 2a) + \dots \dots \dots]$$

Find the velocity as a function of time.

3. Solve the following differential equation –  
 $x''(t) + 4x(t) = e^t$  subject to the initial conditions  $x(0) = 0$  and  $x'(0) = 1$  by application of Laplace transform.
4. Solve the following differential equation by power series solution –  
 $\frac{d^2y}{dx^2} + y = 0$ .

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# JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

## DEPARTMENT OF MATHEMATICS



INTERNAL EXAMINATION – 2023

ACADEMIC YEAR : 2022 - 2023

SEM: II

SUBJECT: MATHEMATICS

PAPER: C3T

**Maximum Marks: 10**

### ANSWER ANY FIVE OF THE FOLLOWING

1. If  $a \in \mathbb{R}$  and  $0 \leq a < \frac{1}{n}$  for every natural number  $n$ , prove that  $a = 0$ .
2. Find  $\text{Sup } A$  and  $\text{Inf } A$ , Where  $A = \left\{ \frac{n+(-1)^n}{n}; n \in \mathbb{N} \right\}$ .
3. Prove that the set  $S = \{x \in \mathbb{R} : \sin x \neq 0\}$  is an open set.
4. Show that a finite set has no limit point.
5. Prove that the set  $\left\{ \frac{1}{m} + \frac{1}{n} : m \in \mathbb{N}, n \in \mathbb{N} \right\}$  is closed in  $\mathbb{R}$ .
6. Show that the  $\{x_n\}$  sequence is a Null sequence, where  $x_n = \frac{n!}{n^n}$ .
7. Use Sandwich Theorem to prove that  
$$\lim \left[ \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(n+n)^2} \right] = 0.$$

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# JHARGRAM RAJ COLLEGE

JHARGRAM - 721 507



## DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION - 2022- 2023

SEM: II SUBJECT: MATHEMATICS PAPER: C 4 T (Differential Equation & Vector Calculus)

Date: 25/07/2023

Maximum Marks: 10

### ANSWER ANY ONE OF THE FOLLOWING

1. (a) Determine the 1<sup>st</sup> three approximated solutions of the following Initial Value Problem (IVP) -

$$\frac{dy}{dx} = 2y, y(0) = 1$$

- (b) Show that the following differential equation -

$$(D^4 - D^3 - 3D^2 + 5D - 2)y = 0, D = \frac{d}{dx}$$

has only two linearly independent solutions of the form  $y = e^{mx}$ . But verify that  $y = e^x, y = xe^x, y = x^2e^x, y = e^{-2x}$  are four linearly independent solutions of the given equation. Hence write down the complete solution.

- (c) If  $\vec{a}$  &  $\vec{b}$  be two non collinear vectors such that  $\vec{a} = \vec{c} + \vec{d}$  where  $\vec{c}$  is a vector parallel to  $\vec{b}$  &  $\vec{d}$  is a vector perpendicular to  $\vec{b}$ , then obtain expressions for  $\vec{c}$  &  $\vec{d}$  in terms of  $\vec{a}$  &  $\vec{b}$ .

4 + 3 + 3

2. (a) Solve the following differential equation by the method of variation of parameters -

$$\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2}y = \log x \quad (x > 0)$$

it being given that  $y = x$  and  $y = 1/x$  are two linearly independent solutions of its reduced equation.

- (b) Prove that  $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a} \vec{b} \vec{c}]^2$

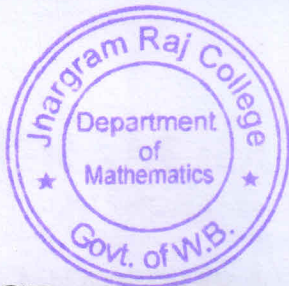
- (c) Solve the following differential equation -

$$\frac{dx}{y^2 + yz + z^2} = \frac{dy}{z^2 + zx + x^2} = \frac{dz}{x^2 + xy + y^2}$$

3 + 2 + 5

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# JHARGRAM RAJ COLLEGE

JHARGRAM - 721 507

## DEPARTMENT OF MATHEMATICS



INTERNAL EXAMINATION - 2023

ACADEMIC YEAR: 2022 - 2023

SUBJECT: MATHEMATICS

SEM: IV

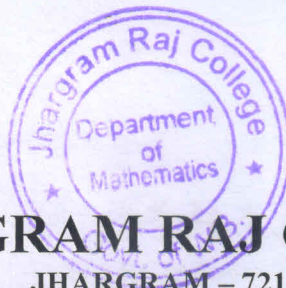
PAPER: C8T

Maximum Marks: 10

### ANSWER ANY FOUR OF THE FOLLOWING

1. A function  $f$  is defined by  $[0,1]$  by  
 $f(0) = 0$   
 $f(x) = (-1)^{r-1}, \frac{1}{r+1} < x \leq \frac{1}{r}, \text{ for } r = 1, 2, 3 \dots$   
Show that  $f$  is integrable on  $[0,1]$
2. Let  $f(x) = [x], x \in [0,3]$ . Evaluate  $\int_0^3 f$ .
3. A function  $f$  is defined by  $[0,1]$  by  
 $f(x) = \sin x, x \text{ is rational.}$   
 $= x, x \text{ is irrational.}$   
Show that  $f$  is not integrable on  $[0, \frac{\pi}{2}]$
4. Let  $f_n(x) = x^{n-1} - x^n, x \in [0,1]$ . Prove that the sequence  $\{f_n\}$  is uniformly convergent on  $[0,1]$ .
5. Let  $f_n(x) = \frac{nx}{1+nx}, x \in [0,1]$ . Show that the sequence  $\{f_n\}$  is not uniformly convergent on  $[0,1]$ .
6. Let  $f_n(x) = x^2 e^{-nx}, x \in [0, \infty]$ . Show that the sequence  $\{f_n\}$  is uniformly convergent on  $[0, \infty]$ .

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## DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION - 2022- 2023

SEM: IV SUBJECT: MATHEMATICS PAPER: C 9 T (Multivariable Calculus)

Date: 24/07/2023

Maximum Marks: 10

### ANSWER ANY ONE OF THE FOLLOWING

1. (a) Let  $f(x) = \begin{cases} \frac{x^2y^2}{x^2+y^2}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0 \end{cases}$

Show that  $f_{xy}(0,0) = f_{yx}(0,0)$  although  $f$  does not satisfy conditions of Schwarz's theorem.

(b) Check that whether  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{x-y}$  exists or not.

(c) Change the order of integration & hence evaluate

$$\int_2^4 \int_{\frac{4}{x}}^{20-4x} (4-y) dy dx$$

5 + 2 + 3

2. (a) Show that  $f(x, y) = \sqrt{|xy|}$  is not differentiable at (0,0).

(b) Compute  $\iint \frac{2x^2+y^2}{xy} dx dy$  taken over the area in the positive quadrant of the  $xy$  plane bounded by the curves  $x^2 + y^2 = h^2, x^2 + y^2 = k^2, y^2 = 4ax, y^2 = 4bx, a, b > 0$

(c) Evaluate  $\iiint z^2 dx dy dz$  over the region E common to the surfaces  $x^2 + y^2 + z^2 = a^2$  &  $x^2 + y^2 = ax$ .

3 + 2 + 5

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## DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2022- 2023  
SEM: IV SUBJECT: MATHEMATICS PAPER: C 10 T (RING THEORY AND LINEAR ALGEBRA - I)

Date: 25/07/2023

Maximum Marks: 10

### ANSWER ANY ONE OF THE FOLLOWING

1. If  $(R, +, \cdot)$  Be a ring such that  $(R, +)$  is a cyclic group, prove that the ring is a commutative ring. Also, deduce that a ring of prime number of elements is always commutative ring.
2. Let  $(R, +, \cdot)$  Be a ring with unity element  $I$  and  $a \in R$ . If there exists a unique element  $b \in R$  such that  $ab = I$ , prove that  $ba = I$  too and  $a$  is a unit.
3. Let  $D$  is an integral domain in which the identity element is the only element which is its own inverse. Prove that the characteristic of  $D$  is 2.
4. Let  $a$  be a fixed element in a ring  $(R, +, \cdot)$  and let  $C(a) = \{x \in R: xa = ax\}$ . Prove that  $C(a)$  is a subring of  $(R, +, \cdot)$ .
5. Let  $\alpha_1, \alpha_2, \alpha_3$  are vectors in real vector space  $V$  such that  $\alpha_1 + \alpha_2 + \alpha_3 = 0$ . Prove that  $L(\alpha_1, \alpha_2) = L(\alpha_2, \alpha_3) = L(\alpha_1, \alpha_3)$ .
6. Let  $V$  be a real vector space with a basis  $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ . Examine if  $\{\alpha_1 + \alpha_2, \alpha_2 + \alpha_3, \dots, \alpha_n + \alpha_1\}$  is also a basis of  $V$ .
7. Determine the linear mapping  $T: R^3 \rightarrow R^3$  that maps the basis vectors  $(0,1,1), (1,0,1), (1,1,0)$  of  $R^3$  to the vectors  $(2,1,1), (1,2,1), (1,1,2)$  respectively. Find also the kernel and image of the transformation.
8. Let  $T$  be a linear operator on a vector space  $V$  over a field  $F$ . Prove that –
  - $Ker(T) \subset Ker(T^2)$
  - $Im(T^2) \subset Im(T)$

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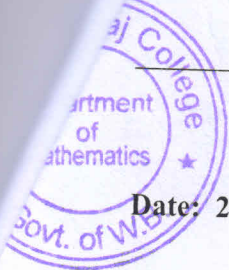


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## DEPARTMENT OF MATHEMATICS



INTERNAL EXAMINATION - 2022-2023  
SEM: IV SUBJECT: MATHEMATICS PAPER: SEC 2 (GRAPH THEORY)

Date: 25/07/2023

Maximum Marks: 5

### ANSWER ANY THREE OF THE FOLLOWING

1. Show that a complete graph with  $n$  vertices consists of  $\frac{n(n-1)}{2}$  edges.
2. Let  $G$  be a simple graph with at most  $2n$  vertices. If the degree of each vertex is at least  $n$ , then show that the graph is connected.
3. Let  $G$  be a graph and  $u, v$  be two vertices of  $G$  such that  $u \neq v$ . If there is a trail from  $u$  to  $v$ , then show that there is a path from  $u$  to  $v$ .
4. Prove that a graph has a circuit if the degree of each vertex is an even positive integer.

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I.A. / Paper - SEC 2 / 2022-23