JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

1st INTERNAL EXAMINATION – 2021- 2022

SEM: IV SUBJECT: MATHEMATICS PAPER: C 9 T (MULTIVARIATE CALCULAS)

Date: 27/04/2022 **Maximum Marks: 10**

ANSWER ANY FIVE OF THE FOLLOWING

01. Verify whether $\lim_{(x,y)\to(0,1)} tan^{-1}\left(\frac{y}{x}\right)$ is exists or not.

02. Find the repeated limits of the function $f(x,y) = \frac{x-y}{x+y}$ at the origin.

03. Investigate the continuity at (0,0) of the function $f(x,y) = \frac{x^3+y^3}{x-y}, x \neq y$
$= 0, \quad x = y$

04. If $f(x,y) = \sqrt{|xy|}$, find $f_x(0,0)$ and $f_y(0,0)$.

05. Show that the function $f(x,y) = |x| + |y|$ is not differentiable at the origin.

06. Show that $z = f(x^2y)$, where f is differentiable, satisfies $x\left(\frac{\partial z}{\partial x}\right) = 2y\left(\frac{\partial z}{\partial y}\right)$.

07. Find all the stationary points of the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

08. In what direction from the point (1,3,2) is the directional derivative of $\Phi = 2xz - y^2$ is maximum? What is the magnitude of this maximum?

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INTERNAL EXAMINATION – 2021- 2022

SEM: VI SUBJECT: MATHEMATICS PAPER: DSE3 (NUMBER THEORY)

Maximum Marks: 10

Date: 07/04/2022

ANSWER ANY FIVE OF THE FOLLOWING

1. For any integer $n > 1$ prove that $\tau(n) \leq 2\sqrt{n}$, where $\tau(n)$ is number of positive divisor of n.

2. The Mangoldt function $\wedge$ is defined by

$$\wedge(n) = \log p \,, \quad n = p^k, p \text{ is a prime and } k \geq 1.$$
$$0, \quad otherwise$$

Prove that $\log_e n = \sum_{d|n} \wedge(d)$

3. Prove that if $n \geq 1$ then $\frac{(2n)!}{(n!)^2}$ is an even integer.

4. Prove that if $gcd\,(a, n) = gcd\,(a-1, n) = 1$, then
$$1 + a + a^2 + \cdots + a^{\varphi(n)-1} \equiv 0 (mod\; n).$$

5. Prove that for $n > 1$, the average of the positive integers less than n and relatively prime to n is $\frac{n}{2}$.

6. Prove that $n | \varphi(2^n - 1), \forall\, n > 1$.

7. If $gcd\,(m, n) = 1, m > 2, n > 2$ then prove that the integer mn has no primitive roots.

8. Prove that if p is an odd prime, then $x^2 \equiv -1 (mod\; p)$ is solvable iff $p \equiv 1 (mod\; 4)$.

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INTERNAL EXAMINATION – 2021- 2022

SEM: VI SUBJECT: MATHEMATICS PAPER: C 14 T (RING THEORY II & LINEAR ALGEBRA II)

Date: 06/04/2022 **Maximum Marks: 10**

ANSWER ANY FIVE OF THE FOLLOWING

1. Let p be an irreducible element of a PID R. Prove that $< p >$ is a non-zero maximal ideal.

2. Find an ideal in the polynomial ring $\mathbb{Z}[x]$ which is not a principal ideal. Hence justify that $\mathbb{Z}[x]$ is not a PID.

3. Prove that 2 & $1 + i\sqrt{5}$ are relatively prime in the integral domain $\mathbb{Z}[i\sqrt{5}.]$

4. Prove that every Euclidean domain is a PID.

5. Suppose that W is a finite dimensional vector space and that $T: V \to W$ is linear. Prove that $N(T^t) = (R(T))^0$ where $S^0 := \{f \in V^* : f(x) = 0 \ \forall \ x \in S\}$ For every subset S of V.

6. Prove that $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is not diagonalizable.

7. Let V be a finite dimensional vector space, and define $\varphi: V \to V^{**}$ by $\varphi(x) = \hat{x}$. Prove that φ is an isomorphism.

8. Prove that annihilating polynomial for a square matrix always exists.

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INTERNAL EXAMINATION – 2021- 2022

SEM: VI SUBJECT: MATHEMATICS PAPER: C 13 T (METRIC SPACE & COMPLEX ANALYSIS)

Date: 06/04/2022 **Maximum Marks: 10**

ANSWER ANY FIVE OF THE FOLLOWING

01. Investigate the convergence of the sequence in (R, d_u) -
$\{x_n\}$, where $x_n = 1 - \frac{1}{n}, n \in N$.
[The symbols have their usual meaning]

02. Let (X, d) be a metric space and $\{x_n\}$ and $\{y_n\}$ are two convergent sequences converges to x and y respectively. Prove that the sequence $\{d(x_n, y_n)\}$ is also converges to $d(x, y)$.

03. Let X be a non – empty set and d_1 & d_2 are two equivalent metrics on X. If $\{x_n\}$ is a Cauchy sequence in (X, d_1) then prove that $\{x_n\}$ is also a Cauchy sequence in (X, d_2).

04. Let (X, d) be a metric space and A, B are two non – empty disjoint closed subsets of X. Prove that $f: (X, d) \to R$ defined as –

$$f(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \in B \end{cases}$$

is a continuous function.

05. Show that the Complex-Valued function $f(z) = 3z + z^2$ is Univalent in Δ, where Δ is an open disk.

06. Discuss the Continuity of Complex-Valued function

$$f(z) = \frac{\bar{z}}{z} \quad z \neq 0$$
$$= 0 \quad z = 0$$

at the origin.

07. Show that $f(z) = Rez$ is nowhere differentiable in $\mathbb{C}$.

08. Let $f(z) = x^2 + iy^2$.Is $f(z)$ is analytic at the origin?

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INTERNAL EXAMINATION – 2021- 2022

SEM: I SUBJECT: MATHEMATICS PAPER: C 1 T (CALCULUS, GEOMETRY & DIFFERENTIAL EQUATIONS)

Date: 22/02/2022 **Maximum Marks: 10**

ANSWER ANY FIVE OF THE FOLLOWING

1. Find $(y_n)_0$, if $y = \sin(a\sin^{-1}x)$.
2. If $y = tan^{-1}x$, prove that $(1+x^2)y_2 + 2xy_1 = 0$ and deduce that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. Hence determine $(y_n)_0$.
3. Solve $e^x cosy dy + (e^x siny + 2x)dx = 0$.
4. Solve $\frac{dy}{dx} + 4xy = 8x$.
5. Choose a new origin (h, k) without changing the direction of the axes such that the equation $5x^2 - 2y^2 - 30x + 8y = 0$ may reduce to the form $aX^2 + bY^2 = 1$.
6. Prove that if the tangent at any point P of a conic meets the directrix in K, the angle KSP is a right angle.
7. Find the centre and the radius of the circle given by $2x - 3y + 6z = 62, x^2 + y^2 + z^2 - 4x + 2y - 2z - 58 = 0$.
8. Evaluate $\int_0^{\frac{\pi}{4}} \sin^4 x dx$ By using Reduction Formula.

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INTERNAL EXAMINATION – 2021- 2022

SEM: I SUBJECT: MATHEMATICS PAPER: C 2 T (ALGEBRA)

Date: 03/03/2022

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING

1. If a,b, and c are three positive real numbers greater than 1, then show that
$$abc + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} > a + b + c + \frac{1}{abc}$$
2. Apply Descarte's Rule of sign to find the nature of the roots of the equation –
$$2x^4 + 3x^2 - 4x - 1 = 0$$
3. Let $k > 1$ & $2^k - 1$ is a prime. If $n = 2^{k-1}(2^k - 1)$ then show that n is a perfect number.
4. If $gcd\ (a, b)\ = 1$ then Prove that $gcd\ (a^2, b^2)\ = 1$.
5. Solve the system of equations
$$x + 2y + z = 1$$
$$3x + y + 2z = 3$$
$$x + 7y + 2z = 1$$
6. Examine if the set S = $\{(x, y, z) \in R^3 : 2x - y + 3z = 0\}$ is a subspace of R^3.
7. Determine k so that the set S = $\{(k, 3,1), (2, k, 0), (1,2,1)\}$ is linearly independent in R^3.
8. Find a basis for the vector space R^3 that contains the vectors $(1, 0, 1)$ and $(1, 1, 1)$.

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