JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR: 2020 - 2021

INTERNAL EXAMINATION – 2021
SEM: II SUBJECT: MATHEMATICS PAPER: C 3 T (REAL ANALYSIS)

Full marks: 10

Answer any five questions **$(5 \times 2 = 10)$**

1) Let S be a non – empty subset of R bounded below and $T = \{-x : x \in S\}$. Prove that the set T is bounded above and $\sup T = -\inf S$.
2) If y be a positive real number show that there exists a natural number m such that $0 < 1/2^m < y$.
3) Let A and B are subsets of R of which A is closed and B is compact. Prove that $A \cap B$ is compact.
4) Let $S = (a, b)$ an open bounded interval. Prove that $S' = [a, b]$. Here S' denote the derived set of S.
5) Prove that a convergent sequence is bounded.
6) Use Sandwich theorem to prove that $lim\left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2}\right] = 0$.
7) Use Cauchy's general principle of convergence to prove that the sequence $\{u_n\}$ where $u_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is not convergent.
8) Prove that $\lim\limits_{n\to\infty} \frac{1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots\frac{1}{2n+1}}{2n+1} = 0$.

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Answer booklets has to emailed to the following address –

jrcmathematics2020@gmail.com

Department of Mathematics
Jhargram Raj College
B.Sc(H), Sem-II Internal Assessment 2020-21
Sub: Mathematics, Course: C4

Full Marks: 10

Answer any One question **$(1 \times 10 = 10)$**

1. (a) Solve by the method of undetermined coefficients: $y_2 - 4y_1 + 4y = x^3e^{2x} + xe^{2x}$.

   (b) Solve by the method of variation of parameters: $(2x+1)(x+1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = (2x+1)^2$.
   It is being given that $y = x \, and \, y = \frac{1}{x+1}$ are two linearly independent solutions of the corresponding homogeneous equation. 5+5

2. (a) Solve: $y_2 - y = 1$, given that y = 0 when x = 0 and $y \to a$ finite limit when $x \to -\infty$

   (b) Solve: $(D^4 + D^3 - 3D^2 - 5D - 2)y = 3xe^{-x}$

   5+5

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JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2021

SEM: III SUBJECT: MATHEMATICS PAPER: C 7 T 7 (NUMERICAL METHODS)

Maximum Marks: 10

ANSWER ANY FIVE OF THE FOLLOWING

1) If 0.333 is the approximate value of $\frac{1}{3}$, then determine the absolute, relative and percentage error.
2) Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its absolute, relative and percentage error.
3) Explain "Bisection Method" is a root bracketing method. Find a positive root of the equation $x^3 + 3\text{x} - 1 = 0$ by bisection method.
4) Suggest a value of constant k, so that the iteration formula $x = x + k(x^2 - 3)$ may converge at a good rate, given that $x = 3$ is a root.
5) Using Newton's backward difference formula, find the value of $e^{-1.9}$ from the following table of values –

x	1.00	1.25	1.50	1.75	2.00
e^{-x}	0.3679	0.2865	0.2231	0.1738	0.1353

6) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, \text{dx}$, given that –

x	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$
$\sqrt{\sin x}$	0	0.5087	0.7071	0.8409	0.9306

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JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR: 2020 - 2021

INTERNAL EXAMINATION – 2021

SEM: IV SUBJECT: MATHEMATICS PAPER: C 8 T (RIEMANN INTEGRATION AND SERIES OF FUNCTIONS)

Maximum Marks: 10

1) A function f is defined on $[0,1]$ by –

$$f(x) = \begin{cases} 0, x = 0 \\ (-1)^{n-1}, \frac{1}{n+1} x \leq \frac{1}{n} \ (n = 1,2,3, \dots\dots) \end{cases}$$

Prove that -

a. The function f is Riemann integrable on $[0,1]$.

b. $\int_0^1 f = \log(4/e)$.

2)

a. If a function f is continuous on a closed and bounded interval $[a,b]$ and $\int_a^b fg = 0$ for every continuous function g on $[a,b]$, prove that $f(x) = 0, \forall x \in [a,b]$.

b. If a function f is integrable on $[a,b]$ and $\int_a^b f^2 = 0$, prove that $f(x) = 0$ at every point of continuity in the closed and bounded interval $[a,b]$.

3) Let $f: [a,b] \to R$ be bounded and monotone increasing on closed and bounded interval $[a,b]$. If P_n be the partition of the closed interval $[a,b]$ dividing into n sub-intervals of equal length, prove that –

$$\int_a^b f \leq U(P_n, f) \leq \int_a^b f + \frac{b-a}{n}\big(f(b) - f(a)\big)$$

Also consider the sequence of partitions $\{P_n\}_n$ and deduce that $\lim_{n\to\infty} U(P_n, f) = \int_a^b f$.

4) Let $\{f_n\}$ and $\{g_n\}$ be sequences of functions convergent uniformly to f and g respectively on $[a,b]$. Prove that the sequence of functions $\{f_n + g_n\}$ converges uniformly to the sum function $f + g$ on $[a,b]$.

5) Show that a sequence of functions $\{f_n\}$ on $[a,b]$ is uniformly convergent to a function f if and only if $\lim_{n\to\infty} M_n = 0$, where $M_n = \sup|f_n(x) - f(x)|$.

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JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR: 2020 - 2021

INTERNAL EXAMINATION – 2021

SEM: IV SUBJECT: MATHEMATICS PAPER: C9T

Full marks: 10

Answer any five questions **$(5 \times 2 = 10)$**

1) Let $f(x,y) = \frac{x^2y^2}{x^2y^2+(x-y)^2}$ if $(x,y) \neq 0$

   $= 0$ if $(x,y) = 0$

   Show that repeated limit exist at the origin but double limit does not exist.

2) Show that the function $f(x,y) = xsin\frac{1}{y} + ysin\frac{1}{x}$, when $xy \neq 0$

   $= 0$ when $xy = 0$

   is continuous at (0,0).

3) If $u = \cos^{-1}\frac{(2x+y)}{(\sqrt{x}+\sqrt{y})}$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}cotu$.

4) Examine the extrema of the function $f(x,y) = x^2 + 3xy + y^2 + x^3 + y^3$.

5) Find the equation of the tangent plane to the surface $z = xy$ at the point (2,3,6).

6) Evaluate $\nabla \times (\frac{\vec{r}}{r^2})$.

7) Prove that the vector $\vec{A} = 3y^4z^2\hat{\imath} + 4x^3z^2\hat{\jmath} - 3x^2y^2\hat{k}$ is solenoidal.

8) Find the work done in a moving particle in the forced field $\vec{F} = (2y+3)\hat{\imath} + xz\hat{\jmath} + (yz - x)\hat{k}$ along the straight line joining (0,0,0) to(2,1,1).

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DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – IV, INTERNAL ASSESSMENT, 2020-21
Sub: MATHEMATICS, Course – C10

Full Marks: 10

Answer any One question: **$(1 \times 10 = 10)$**

1. (a) Find $U\left(\mathbb{Z}\left[\sqrt{-5}\right]\right)$.

(b) Show that it is impossible to have an integral domain with 6 elements.

(c) Let R be a commutative ring with unity and I be an ideal in R. Show that $rad(I) = \{a : a \in R, a^n \in I \; for \; some \; n\}$ is an ideal in R. What is $rad(Zero \; ideal)$?

(d) Let V denote the vector space consisting of all upper triangular $n \times n$ matrices and let W_1 denote the subspace of V consisting of all diagonal matrices. Show that $V = W_1 \oplus W_2$ where $W_2 = \{A \in V : A_{ij} = 0 \; when \; i \geq j\}$.

$2 + 2 + 2 + 4$

2. (a) Show that R/N has no nilpotent elements except zero where N is the nil radical of R.

(b) For a fixed $a \in \mathbb{R}$, determine the dimension of the subspace of $P_n(\mathbb{R})$ defined by $\{f \in P_n(\mathbb{R}) : f(a) = 0\}$.

(c) Let V be a vector space and $L(S) = V$. If β is a maximal linearly independent subset of S, then prove that β is a basis for V.

(d) Let R be a ring with unity. Prove that if $char \; R = 0$ then $\mathbb{Z}$ is isomorphic to a sub ring of R.

$2 + 3 + 3 + 2$

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – V , INTERNAL ASSESSMENT, 2020-21
Sub: MATHEMATICS, Course – C11

Full Marks: 10

Answer any one question: $\mathbf{1 \times 10 = 10}$

1. (a) The velocities of a point parallel to the axes of x and y are $u + \omega y$ & $v + \omega^{/} x$ respectively when $u, v, w, \omega^{/}$ are all constants. Show that the path of the points is a conic.

   (b) If the radial velocity is proportional to the transverse velocity find the path in polar coordinates.

   (c) Prove that a planet has only a radial acceleration towards the sun.

   (d) A particle is projected along the inner surface of a rough sphere and is acted on by no forces. Write down the equations of motion with proper explanation and picture.

   $\mathbf{3 + 2 + 3 + 2}$

2. (a) Classify the equation when it will be parabolic, elliptic, hyperbolic $(1 - x^2)r - 2xys + (1 - y^2)t + 2px + 6x^2yq - 6z = 0$.

   (b) Write down the characteristic equations of $r = x^2t$.

   (c) Find the surface satisfying $t = 6x^2y$ containing two lines $y = 0 = z$ & $y = 2 = z$.

   (d) State Cauchy- Kowalewskaya Theorem.

   $\mathbf{2 + 3 + 4 + 1}$

Department of Mathematics

Jhargram Raj College

B.Sc(H), Sem – V Internal Assessment – 2021

Sub: Mathematics, Course: CC – 12

Full Marks: 10 **Date:16.03.2021**

Answer any one question.

1. **Define inner automorphism of a group G. Show that (a) $Inn(G)$ is a normal subgroup of $Aut(G)$.**
   (b) $G/_{Z(G)} \simeq Inn(G)$ where Z(G) is the centre of G and hence find the order of $Inn(S_3)$.

2. **Let G be a finite group. Then show that $|G| = |Z(G)| + \sum_{a \notin Z(G)} [G: C(a)]$ where $Z(G)$ is the centre of G and hence find the class equation for S_3.**

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JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR: 2020 - 2021

INTERNAL EXAMINATION – 2021

SEM: VI SUBJECT: MATHEMATICS PAPER: C 13 T (METRIC SPACES & COMPLEX ANALYSIS)

Full Marks: 10

Answer any five questions (2×5=10)

1. Let $\{x_n\}$ and $\{y_n\}$ be two Cauchy sequences in a metric space (X,d). Prove that $d(x_n,y_n) \to 0$ as $n \to \infty$ if and only if the sequence z_n in (X,d), where $z_{2n} = \mathrm{x_n}, \mathrm{z_{2n-1}} = \mathrm{y_n}$, for all $n \in N$, is a Cauchy sequence.
2. Let A and B be two closed subsets in a metric space (X,d_1) such that $X = A \cup B$, and let $f: A \to (Y,d_2)$ and $g: B \to (Y,d_2)$ be continuous mappings such that $f(x) = g(x), \forall x \in A \cap B$. Show that the combination mapping $h: (X,d_1) \to (Y,d_2)$, defined by $h(x) = \begin{cases} f(x), x \in A \\ g(x), x \in B \end{cases}$, is continuous.
3. If an open (respectively closed) set U in a metric space (X,d) can be expressed as a union of two separated sets A and B, show that each of A,B is open (respectively closed).
4. Suppose that f is analytic in a domain D. If the range $f(D)$ lies in a circle then prove that f is constant.
5. Let γ be a closed contour and z be a point not on γ, then prove that the Winding number of γ with respect to z is an integer.
6. Suppose f *and* g are entire functions; g is never zero and $|f(z)| \leq |g(z)|\ \forall\ z$. Show that there is a constant c such that $f(z) = c\ g(z)$.
7. Let f be analytic in $|z| < 5$, suppose that $|f(s)| \leq 10$ for all points on the circle $|s-1| = 3$. Find a bound for $|f^3(0)|$.

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JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR: 2020 - 2021

INTERNAL EXAMINATION – 2021
SEM: VI SUBJECT: MATHEMATICS PAPER: C14T

Full marks: 10

Answer any five questions **($5 \times 2 = 10$)**

1) Let R be a ring with 1.Show that $R[x]/\langle x\rangle \simeq R$.
2) In the ring Z_8, show that $[1]+[2]x$ is a unit.
3) Show that $Z[x]$ is not a PID.
4) Show that $[4]$ and $[6]$ are associates in Z_{10}.
5) Let R be an integral domain and p be a prime element in R. Then show that p is irreducible.
6) In $Z[i]$, show that 3 is prime element but 5 is not prime element.
7) Show that the polynomial $2x^4+6x^3-9x^2+15$ is irreducible in $Z[x]$.
8) Find all irreducible polynomials of degree 2 over the field Z_3.

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DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT, 2020-21
Sub: MATHEMATICS, Course:CC-1

Full Marks: 10

Answer any five questions: $(\mathbf{5 \times 2 = 10})$

1. Find y_n, if $y = x^{n-1} \log x$.
2. If c is a point of inflection for f then prove that either $f^{//}(c) = 0$ or $f^{//}(x)$ does not exists.
3. Solve $(x^2 - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.
4. Solve $(1 + x^2)\frac{dy}{dx} + (1 - x)^2 y = xe^{-x}$.
5. Show that the semi latus rectum of a conic is a harmonic mean between the segments of any focal chord.
6. Find the equation of the smallest sphere passing through the points $(1,0,0), (0,1,0)$ & $(0,0,1)$.
7. If P be a variable point such that its distance from the XZ plane is always equal to its distance from the y axis then show that the locus of P is a cone.
8. Evaluate $\int_0^{\frac{\pi}{4}} tan^6 x dx$, By using Reduction Formula.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT, 2020-21
Sub: MATHEMATICS, Course:CC-2

Full Marks: 10

Answer any five questions: $(\mathbf{5 \times 2 = 10})$

1. Determine $[2(\cos 30^o + i \sin 30^o)]^5$ by application of De Moivre's Theorem. . Write the answer in rectangular form.
2. Let $a_1, a_2, \ldots\ldots, a_n$ be n positive real numbers whose product is 1. Prove that $(1+a_1)(1+a_2)(1+a_3)\ldots\ldots\ldots(1+a_n) \geq 2^n$.
3. If n be an even integer then prove that $\varphi(2n) = 2\varphi(n)$.
4. Prove that $[x] + [-x] = 0\ or - 1$ according as $x \in \mathbb{Z}\ or\ not$.
5. Find the fully reduced normal form of the matrix $\begin{pmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.
6. Determine the condition for which the system of equations

$$\begin{aligned} x + 2y + z &= 1 \\ 2x + y + 3z &= b \\ x + ay + 3z &= b + 1 \end{aligned}$$

has many solutions.

7. Examine if the set $S = \{(x, y, z) \in R^3 : 2x - 3y + 5z = 0\}$ is a subspace of R^3.
8. Show that the set of vectors $\{(1,2,3,0), (2,3,0,1), (3,0,1,2)\}$ are linearly independent in R^4.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2020-21
Sub: MATHEMATICS, Course – C5

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{5 \times 2 = 10})$

1. Let $f:[a,b] \to \mathbb{R}$ be continuous in $[a,b]$. Let $x_1, x_2, \dots, x_n$ be points of $[a,b]$. Show that there exist a point $\theta \in [a,b]$ such that $f(\theta) = \frac{1}{n}\sum_{i=1}^{n} f(x_i)$.

2. Suppose f has the property that $|f^{/}(x)| < 1 \; \forall \; x \in (0,1)$ & f be continuous at $x = 0$ & $x = 1$. Show that the sequence $\{f\left(\frac{1}{n}\right)\}$ has a limit.

3. Let $f: \mathbb{R} \to \mathbb{R}$ be infinitely differentiable function. Suppose that for some $n \in \mathbb{N}, f(1) = f(0) = f^{/}(0) = \cdots = f^{n}(0)$. Show that $f^{n+1}(x) = 0$ for some $x \in (0,1)$.

4. From the definition prove that $\lim_{x\to 0} e^{\frac{-1}{x^2}} = 0$.

5. Show that $|x|$ has no Maclaurin's series expansion.

6. Prove that in a metric space (X,d) for $A \subset X$, $\partial A = \partial(X \backslash A)$.

7. Prove that in a metric space (X,d) for $A \subset X, \bar{A}$ is the smallest closed subset of X containing A.

8. Prove that $d_1(x,y) = \min\{1, d(x,y)\}$ is a metric if d is a metric.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT, 2020-21
Sub: MATHEMATICS, Course:CC-6

Full Marks: 10

Answer any five questions: $(\mathbf{5 \times 2 = 10})$

1. If each element in a group be its own inverse then prove that the group is abelian.
2. Find the all elements of order 5 in the group $(Z_{30}, +)$.
3. Prove that centralizer of an element in a group G is a subgroup of G.
4. Prove that in a cyclic group of even order, there is exactly one element of order 2.
5. Let G be a group and $a \in Z(G)$. Prove that $\langle a \rangle$ is a normal subgroup of G.
6. Prove that a commutative group of order 10 is cyclic.
7. Find all homomorphisims from the group $(Z_8, +)$ to the group $(Z_6, +)$.
8. Show that the groups $(Q, +)$ and $(R, +)$ are not isomorphic.

Department of Mathematics
Jhargram Raj College
B.Sc(H), Sem – V Internal Assessment – 2021
Sub: Mathematics, Course: DSE – 1

Full Marks: 10 **Date:16.03.2021**

Answer any one question.

1. **Find the dual of the following problem and solve the dual problem. Also find the solution of the primal problem from the dual.**

Maximize Z = $5x_1 - 2x_2 + 3x_3$,
Subject to

$$2x_1 - 2x_2 + x_3 \geq 2,$$
$$3x_1 - 4x_2 \leq 3,$$
$$x_2 + 3x_3 \leq 5,$$
$$x_1, x_2, x_3 \geq 0$$

2. (a) Find the optimal solution of the transportation problem

	D_1	D_2	D_3	D_4	
O_1	5	3	6	4	30
O_2	3	4	7	8	15
O_3	9	6	5	8	15
	10	25	18	7	

(b) Find the optimal assignment for the assignment problem with the following cost matrix

	I	II	III	IV	V
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

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DEPARTMENT OF MATHEMATICS

INTERNAL EXAMINATION – 2021

SEM: V SUBJECT: MATHEMATICS PAPER: DSE – II (PROBABILITY & SATISTICS)

Date: 16/03/2021 **Maximum Marks: 10**

ANSWER ANY TWO OF THE FOLLOWING

1) A box of 250 transistors all equal in size, shape etc. 100 of them are manufactured by A, 100 by B and the rest by C. The transistors from A, B, C are defective by 5%, 10% and 25% respectively.

a) A transistor is drawn at random. What is the probability that it is defective?

b) Find the probability that it is made by A given that it is defective.

2) A fair coin is tossed twice. The random variable X is defined as the number of times heads shows up. Define the events (X = 0), (X = 1) and (X = 2) and determine the probability mass function of the random variable X.

3) The probability density function of a continuous random variable X is given by –

$$f(x) = \begin{cases} cx^2, 0 \le x \le 1 \\ 0, otherwise \end{cases}$$

Calculate the unknown c and also determine $P(|X|^2 \le 0.5)$.

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DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – VI , INTERNAL ASSESSMENT, 2020-21
Sub: MATHEMATICS, Course – DSE3

Full Marks: 10
Answer any One question: **($1\times 10 = 10$)**

1. (a) If $\varphi(n)|(n-1)$ then prove that n is a square free integer.

   (b) If $\gcd(m,n) = 1$ prove that $m^{\varphi(n)} + n^{\varphi(m)} \equiv 1(mod\ mn)$.

   (c) Prove that $[x] + [-x] = 0\ or -1$ according as $x \in \mathbb{Z}\ or\ not$.

   (d) If $a, b \in \mathbb{N}$ such that $\frac{a+1}{b} + \frac{b+1}{a} \in \mathbb{N}$ then prove that gcd $(a, b) \leq \sqrt{a+b}$

   $\mathbf{2+2+2+4}$

2. (a) For any integer $n \geq 3$ Show that $\sum_{k=1}^{n} \mu(k!) = 1$ where μ is the Mobius function.

   (b) Let r be a primitive root of the integer n . Prove that r^k is a primitive root of n iff $\gcd(k, \varphi(n)) = 1$.

   (c) If n is a positive integer , then prove that – –

   (i) $\sum_{n=1}^{N} \tau(n) = \sum_{n=1}^{N} [\frac{N}{n}]$

   (ii) $\sum_{n=1}^{N} \sigma(n) = \sum_{n=1}^{N} n\,[\frac{N}{n}]$

   (d) Find the average of all positive integers less than n and prime to n. –

   (e) Prove that there are primitive roots for $2p^k$ where $p\ is\ an\ odd\ prime\ and\ k \geq 1$.

   $\mathbf{2+2+2+2+2}$

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JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR: 2020 - 2021

INTERNAL EXAMINATION – 2021

SEM: VI SUBJECT: MATHEMATICS PAPER: DSE – 4T (MATHEMATICAL MODELLING)

Maximum Marks: 10

1) Let $f(t)$ has the laplace transform $F(s)$. Prove that for any constant c the laplace transform of the function $e^{ct}f(t)$ is $F(s-c)$. [1^{st} shifting property]
2) Determine the following –
   $L(3e^{-2t}\cos 5t)$.
3) Prove that laplace transform is a linear operation, that is for any two functions $f(t), g(t)$ whose laplace transform exists and for any constants a, b –
   $L\big(af(t)+bg(t)\big) = aL\big(f(t)\big) + bL\big(g(t)\big)$
4) Determine the laplace transform f the following functions –
   a. $f(t) = \cosh t . \cos t$
   b. $f(t) = \frac{1}{t}(1-\cos t)$
5) Determine the inverse laplace transform of the following function –
   $F(s) = \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s^4}$.

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JHARGRAM RAJ COLLEGE

JHARGRAM – 721 507

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR: 2020 - 2021

INTERNAL EXAMINATION – 2021

SEM: II SUBJECT: MATHEMATICS PAPER: GE – II T (ALGEBRA)

Full marks: 10

Answer any five questions. **(5×2=10)**

1. If $x_j = \cos\theta_j + i\sin\theta_j$, $(j = 1,2,3, \ldots.., n)$ show that –

$$x_1 x_2 \ldots\ldots x_n + \frac{1}{x_1 x_2 \ldots\ldots x_n} = 2\cos(\theta_1 + \theta_2 + \cdots.. + \theta_n)$$

2. If α, β, γ be the roots of the equation $x^3 - qx + r = 0$, find the equation whose roots are $\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} - \frac{1}{\gamma^2}\right), \left(\frac{1}{\beta^2} + \frac{1}{\gamma^2} - \frac{1}{\alpha^2}\right), \left(\frac{1}{\gamma^2} + \frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)$.
3. If $A \cup B = A \cup C$ and $A \cap B = A \cap C$ $prove$ $that$ $B = C$.
4. Prove that $2^n - 5^n - 6^n + 9^n$ is divisible by 12 for all $n \in \mathbb{N}$.
5. Determine the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 3 & 6 & 6 & 3 \end{pmatrix}$.
6. Determine the conditions for which the system of equations has many solutions.

$$x + y + z = b$$
$$2x + y + 3z = b + 1$$
$$5x + 2y + az = b^2$$

7. Use cayley-Hamilton theorem to find A^{100}, where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.
8. Find the eigen values and the corresponding eigen vectors of the real matrix $\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$.

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Answer booklets has to emailed to the following address –

jrcmathematics2020@gmail.com

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT, 2020-21
Sub: MATHEMATICS, Course – GE1

Full Marks: 10
Answer any five questions: $(\mathbf{5 \times 2 = 10})$

1. Find the asymptotes parallel to the coordinate axes of the curve –
   $4x^2 + 9y^2 = x^2y^2$.
2. Evaluate the following limit –
   $\lim_{x\to 0} \frac{sin^2 x}{1-\cos x}$, by application of **L'Hospital's Rule.**
3. If $J_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$ then prove that $J_n + n(n-1)J_{n-2} = n(\frac{\pi}{2})^{n-1}$.
4. Prove that the length of one arc of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ is $8a$.
5. Find the equation to the curve $9x^2 + 4y^2 + 18x - 16y = 11$ referred to parallel axes through the point (-1, 2).
6. Determine the nature of the conic $x^2 - 6xy + y^2 - 4x - 4y + 12$.
7. Solve $(y^2 + 2x)dx + 2xydy = 0$.
8. Solve $(x^2 - y)dx + (y^2 - x)dy = 0$.

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DEPT. OF MATHEMATICS
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B.Sc(H) Sem – III , INTERNAL ASSESSMENT, 2020-21
Sub: MATHEMATICS, Course – SEC-1

Full Marks: 5

Answer any two questions: $(\mathbf{2 \times 2.5 = 5})$

1. Show that $p \leftrightarrow q$ is logically equivalent to $p \to q$ and $q \to p$.

2. Show that $\big((p \to q) \wedge (q \to r)\big) \to (p \to r)$ is a tautology.

3. Let $p(x, y)$ denotes"$x + y = 5$". Assume that domain is the set of real numbers.
   Find the truth value of the following (a) $\forall x\ \exists y\ \ p(x, y)$. $(b) \exists y\ \ \forall x\ \ p(x, y)$.

Department of Mathematics
Jhargram Raj College
B.Sc(H), Sem-IV Internal Assessment 2020-21
Sub: Mathematics, Course: SEC-2

Full Marks: 5

Answer any two questions: **(2.5**$\times$ 5 = 5)

1. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{n}$.
2. Show that in a graph, the total number of odd degree vertices is even.
3. Prove that in a Euler graph all the vertices are of even degree.

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