DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – I , INTERNAL ASSESSMENT-2nd , 2019-20

Sub: MATHEMATICS, Course – C1

...s: 10 Time: 30 m.

...any five questions: $(2 \times 5 = 10)$

1. Prove that, if $a > 0$, then $\lim_{x\to 0+0} \frac{x}{a}\left[\frac{b}{x}\right] = \frac{b}{a}$ and $\lim_{x\to 0+0} \left[\frac{x}{a}\right]\frac{b}{x} = 0$, where $[x]$ is the greatest integer in x but not greater then x. Discuss the left hand limit of these functions.
2. Examine the asymptotes, if any, parallel to the Y – axis of the curve $x^2y^2 - 9x^2 + 2 = 0$
3. Find the length of the circumference of the circle $x^2 + y^2 = 16$.
4. Find the area of the Cardioide $\boldsymbol{r} = \boldsymbol{a}(1 - cos\,\theta)$.
5. Show that the straight line $r\cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$ if $(l\cos\alpha - ep)^2 + l^2 sin^2\alpha = p^2$.
6. Find the angle between the lines in which $x - 3y + z = 0$ cuts the cone $x^2 - 5y^2 + z^2 = 0$.
7. Find the equations of the generators of the hyperboloid $x^2 - y^2 = 2z$ which pass through the point $(5,3,8)$.
8. Solve: $\frac{dy}{dx} + \frac{y}{x}\ln y = \frac{y}{x^2}(\ln y)^2$.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – I , INTERNAL ASSESSMENT-2nd , 2019-20

Sub: MATHEMATICS, Course – C2

29/11/2019

Full Marks: 10 Time: 30 m.

Answer any five questions: $(2 \times 5 = 10)$

1. Examine if the set $S = \{(x,y,z) \in \mathbb{R}^3 : xy = z\}$ is a subspace of $\mathbb{R}^3$.
2. Find a basis for the vector space $\mathbb{R}^3$, that contains the vectors (1,0,1) and (1,1,1).
3. Find the dimension of the subspace S of $\mathbb{R}^3$ defined by $S = \{(x,y,z) \in \mathbb{R}^3 : 2x + y - z = 0\}$.
4. Let $f: A \to B$ be a mapping. A relation ρ on A is defined as $x \,\rho\, y$ *iff* $f(x) = f(y)$. Prove that ρ is an equivalence relation.
5. Let p & q are distinct primes, $a \in \mathbb{Z}$. Prove that $a^{pq} - a^p - a^q + a$ is divisible by pq.
6. Find the remainder when $1^3 + 2^3 + \cdots + 99^3$ is divided by 3.
7. If a, b, c, d be positive real numbers, each less than 1, prove that $8(abcd + 1) > (a+1)(b+1)(c+1)(d+1)$.
8. If $\alpha, \beta, \gamma, \delta$ are the roots of the polynomial equation $x^4 + px^3 + qx^2 + rx + s = 0$, prove that $\sum \alpha^2\beta = 3r - pq$.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – V, INTERNAL ASSESSMENT-2nd, 2019-20

Sub: MATHEMATICS, Course – C12

Full Marks: 10

Answer any five questions:

Time: 30 m.

($2 \times 5 = 10$)

1. Let H be a subgroup of order 11 and index 4 of a group G. Show that H is a normal subgroup of G.
2. Find the class equation for S_3.
3. Let G be a finite group that has only two conjugate classes. Show that $|G| = 2$.
4. Show that A_4 has no subgroup of order 4.
5. Let G be a noncommutative group of order p^3, p a prime. Prove that $|Z(G)| = p$.
6. How many elements of order 7 are there in a group of order 28?
7. Show that every commutative group of order 36 contains an element of order 6.
8. Prove cayley's theorem by using extended cayley's theorem.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – V , INTERNAL ASSESSMENT-2nd , 2019-20

Sub: MATHEMATICS, Course – DSE1

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Full Marks: 10 Time: 30 m.

Answer any five questions: $(2 \times 5 = 10)$

1. What are the characteristics of the standard form of a linear programming problem?
2. Define slack variable with an example.
3. Define surplus variable with an example.

4. Solve graphically the following LPP
   Maximize: $Z = x - 3y$
   Subject to: $5x + y = 30$; $4x + 3y \geq 12$; $y \leq 5, x, y \geq 0$.

5. Solve the following LPP graphically
   Maximize: $Z = 2x_1 + x_2$
   Subject to: $4x_1 + 3x_2 \leq 12$; $4x_1 + x_2 \leq 8$, $x_1, x_2 \geq 0$.

6. What is redundant constraint? Give an example.
7. Show that $\{X = (x, y): |x| \leq 2\}$ is a convex set.
8. Show that $x_1 = 5$; $x_2 = 0$; $x_3 = -1$ is a basic solution of the system of equations
   $x_1 + 2x_2 + x_3 = 4$ & $2x_1 + x_2 + 5x_3 = 5$.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc. (Honours) Sem. - V, 2nd INTERNAL ASSESSMENT, 2019-20

Sub: MATHEMATICS, Paper - DSE2

Full Marks: 10 **Time**: 30 m.

Answer any five of the following questions: **(5× 2 = 10)**

01. If A and B are two mutually exclusive events connected to a random experiment say E and $P(A \cup B) \neq 0$, then prove that $P\left({}^{A}/_{(A \cup B)}\right) = \frac{P(A)}{P(A)+P(B)}$.

02. Prove that the distribution function of a random variable connected to a random variable say X is left discontinuous function. A coin is tossed. Write down its probability mass distribution and also calculate its distribution function. Verify the above result over the derived distribution function.

03. A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn and are found to be white. What is the probability that all are white?

04. The random variable X has the following distribution function -

$$P(X = k) = 2^{-k}, k = 1,2,3, \ldots.$$

Show that, $E(X) = Var(X) = 2$.

05. If X is uniformly distributed over $[1,2]$, find U so that $P\big(X > U + E(X)\big) = \frac{1}{6}$.

06. The distribution of a random variable X is given by $P(X = -1) = \frac{1}{8}, P(X = 0) = \frac{3}{4}, P(X = 1) = \frac{1}{8}$. Verify Tchebycheff's inequality for the above mentioned distribution.

07. Let T_1 and T_2 be two unbiased estimators of the parameter θ. Under what condition $aT_1 + bT_2$ will be an unbiased estimator of the said parameter θ?

08. Prove that the maximum likelihood estimate of the parameter α of a population having density function $\frac{2}{\alpha^2}(\alpha - x), 0 < x < \alpha$ for a sample of unit size is $2x$, x being the sample value. Show also that the estimate is biased.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc.(H) Sem. – III , INTERNAL ASSESSMENT-2nd , 2019-20

Sub: MATHEMATICS, Course – C5

Full Marks: 10 — Time: 30 m.

Answer any five questions: $(2 \times 5 = 10)$

1. Show that the equation $x \ln x = 3 - x$ has at least one root in $(1,3)$.
2. Is there a function F such that $F'(x) = f(x)$ in $[-1,1]$ where $f(x) = \begin{cases} 0, -1 \le x \le 0 \\ 1, 0 < x \le 1. \end{cases}$
3. Using Maclaurin's series to show that $\sin x > x - \frac{x^3}{6}, 0 < x < \frac{\pi}{2}$.
4. Using LMVT to prove that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}, 0 < a < b < 1$.
5. Prove that f is discontinuous at all irrational points where $f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ 1 - x, & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$
6. Using $\partial A = \bar{A} \cap \overline{(X \setminus A)}$ Prove that $int\ A \cup ext\ A \cup \partial A = X$.
7. Prove that $int\ A$ is the largest open set contained in A. Hence prove that interior operator from $P(X) to\ P(X)$ is an idempotent one.
8. Prove that $\bar{A} = \{x \in X : d(x, A) = 0\}$ & $int(X \setminus A) = \{x \in X : d(x, A) > 0\}$.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – III , INTERNAL ASSESSMENT-2nd, 2019-20

Sub: MATHEMATICS, Course – C6

Full Marks: 10 Time: 30 m.

Answer any five questions: $(2 \times 5 = 10)$

1. Prove that a group of order 27 must have a subgroup of order 3.
2. Let H be a subgroup of a group G and $[G : H] = 2$ then show that H is a normal in G.
3. Prove that if a group G has a unique subgroup H of order 2019 then show that H is a Normal in G.
4. If H be a subgroup of a commutative group G then prove that the quotient group G/H is commutative.
5. Find all homomorphisms from the group $(\mathbb{Z}_6,+)$ to $(\mathbb{Z}_4,+)$.
6. Verify that whether the groups $(\mathbb{Z}_6,+)$ and S_3 are isomorphic or not.
7. Let G be a group of order 9 and H be a group of order 6. Show that there does not exist a homomorphism of G onto H.
8. Find the Centre of Dihedral group of order 8.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc. (Honours) Sem. - III, 2nd INTERNAL ASSESSMENT, 2019-20

Sub: MATHEMATICS, Paper- C 7 T

Full Marks: 10 **Time**: 30 m.

Answer any five of the following questions: $(5\times 2 = 10)$

01. Find a polynomial of least degree which attains the prescribed values at the given points –

x:	-2	-1	0	1	2
$f(x)$:	6	0	2	0	6

02. Define the 1st order forward difference operator (Δ) and the shift operator (E). Establish the relation between them. Hence or otherwise prove that $\left(\frac{\Delta^2}{E}\right)x^3 = 6x$.

03. Find $f(1.02)$ given that –

x:	1.00	1.10	1.20	1.30
$f(x)$:	0.8415	0.8912	0.9320	0.9636

04. Prove that the 3rd order divided difference of a polynomial of degree 3 is constant.

05. Evaluate the 4th order divided difference for equispaced set of arguments.

06. Evaluate $\int_0^5 \frac{dx}{1+x}$, by "Trapezoidal Rule" taking the constant step length as 1.

07. "**Bisection Method**" for determination of the root of a *non – linear or transcendental equation* is a "**Root Bracketing Method**". Explain.

08. When a system of linear algebraic n equations is said to be "**Diagonally Dominant**"?

09. Prove that $\Delta^n x^{(n)} = n!\,h^n$, h is the constant step length.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – III , INTERNAL ASSESSMENT-2nd , 2019-20

Sub: MATHEMATICS, Course – SEC 1

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Full Marks: 5

Time: 20 m.

Answer any two questions:

$(2 \times 2.5 = 5)$

1. Assuming that p & r are false and that q & s are true find the truth value of the proposition $\left(s \to (p \wedge \bar{r})\right) \wedge \left(\left(p \to (r \vee q)\right) \wedge s\right)$
2. Given that p: *Today is Monday*, q: *It is raining*, r: *It is hot*, then express the proposition $\bar{p} \to (q \vee r)$ in words.
3. $p(x, y)$ is a propositional function $x \geq y$. Where domain of discourse is the set of all positive integers. Find the truth value of $\forall x\, \exists y\, p(x, y)$.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – V , INTERNAL ASSESSMENT-2nd , 2019-20

Sub: MATHEMATICS, Course – C11

Full Marks: 10 **Time: 30 m.**

Answer any five questions: **$(2 \times 5 = 10)$**

1. Classify $u_{xx} + u_{yy} + u_{zz} = 0$
2. State Cauchy- Kowalewskaya Theorem.
3. Consider the following Cauchy problem of an infinite string with initial condition
$$u_{tt} - c^2 u_{xx} = 0, x \in \mathbb{R}, t > 0$$
$$u(x,0) = f(x), x \in \mathbb{R} \,\&\, u_t(x,0) = g(x), x \in \mathbb{R}.$$
Write down the corresponding characteristic equation and the integrals.
4. Consider the Cauchy problem for $u_{tt} = c^2 u_{xx} + h^*(x,t)$ with initial conditions$u(x,0) = f(x),\ u_t(x,0) = g^*(x)$. Show that by the coordinate transformation $y = ct$ the above problem reduced to $u_{xx} - u_{yy} = h(x,y); u(x,0) = f(x), u_y(x,0) = g(x)$ $where$ $h(x,y) = -\frac{h^*(x,t)}{c^2}, g(x) = \frac{g^*(x,t)}{c}$.
5. A spherical drop of liquid falling freely in a vapour acquires mass by condensation at a constant rate k. Show that the velocity after falling from rest in time t is $\frac{1}{2}gt\left(1 + \frac{M}{M+kt}\right)$.
6. A smooth parabolic tube is placed vertex downwards, in a vertical plane. A particle slides down the tube from rest under gravity. Write down the equation of motion along the tangent.
7. A point moves along the arc of a cycloid in such a manner that the tangent as it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude
8. A heavy particle slides down a rough cycloid of which the coefficient of friction is μ. Its base is horizontal & vertex downwards. Write down the equations of motion.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – C1

Marks: 10 **Time: 30 m.**

Answer any five questions: **(2 × 5 = 10)**

1. Starting from $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ obtain the expressions for $\frac{d^2x}{d^2y}$ & $\frac{d^3x}{d^3y}$.
2. If $y = \frac{x^3}{x^2-1}$, then prove that $(y_n)_0 = \begin{cases} 0, & \text{if } n \text{ is even;} \\ -n!, & \text{if } n \text{ is odd;} \end{cases}$ $n > 1$.
3. Evaluate $\int tan^5 x\, dx$, using reduction formula.
4. Evaluate $\int_0^{\frac{\pi}{2}} sin^{10} x\, dx$, using Walli's formula.
5. Show that the spheres $x^2 + y^2 + z^2 - 2x + y - 3z + 4 = 0$ & $x^2 + y^2 + z^2 - 5x - 6y + 2z - 5 = 0$ cut orthogonally.
6. If under an orthogonal transformation the expression $ax^2 + 2hxy + by^2 = 0$ changes to $AX^2 + 2HXY + BY^2 = 0$ then Show that $ab - h^2 = AB - H^2$.
7. Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, x + y + z = 3$ as the great circle.
8. Solve: $(1 + x^2)\frac{dy}{dx} + (1 - x)^2 y = xe^{-x}$.

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DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – C2

Full Marks: 10 **Time: 30 m.**

Answer any five questions: **($2 \times 5 = 10$)**

1. Determine the conditions for which the system of equations has many solutions.

$$x + y + z = b$$
$$2x + y + 3z = b + 1$$
$$5x + 2y + az = b^2$$

2. Examine the nature of intersection of the triad of planes.

$x + y - z = 3, 5x + 2y + z = 1, 2x + 2y - 2z = 1;$

3. Use Cayley-Hamilton theorem to find A^{-1}, where A=$\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$.
4. If λ be an eigen value of a real orthogonal matrix A. Prove that $\frac{1}{\lambda}$ is also an eigen value of A.
5. If n be an integer, prove that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$.
6. Prove that if n be composite then $2^n - 1$ is composite.
7. Prove that $(A \cup B)^c = A^c \cap B^c$.
8. Prove that the roots of the equation are all real –

$\frac{1}{x+a_1} + \frac{1}{x+a_2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{x+a_n} = \frac{1}{x}$, where $a_j, j = 1,2,3, \ldots\ldots, n$ are all real positive numbers.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2019-20

Sub: MATHEMATICS, Course – C5

Full Marks: 10

Time: 30 m.

Answer any five questions:

$(2 \times 5 = 10)$

1. Prove that in a metric space (X, d) $|d(x,y) - d(a,b)| \leq d(a,x) + d(b,y)\ \forall\, x, y, a, b \in X$.
2. Prove that (l_p, d) *with* $p \geq 1$ is a metric space where $d(x,y) = (\sum_{n=1}^{\infty} |x_n - y_n|^p)^{\frac{1}{p}}$ where $x = \{x_n\}, y = \{y_n\}$.
3. Prove that every open subset of a discrete metric space is open.
4. Draw $B_d((0,0),1)$ where $d(x,y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$ where $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.
5. Let $f: (-1,1) \to \mathbb{R}$ be continuous at 0.If $f(x) = f(x^2)\ \forall\, x \in (-1,1)$. Prove that $f(x) = 0\ \forall\, x \in (-1,1)$.
6. Let $f: \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$. Prove that for every open subset G of $\mathbb{R}$ $f^{-1}(G)$ is open in $\mathbb{R}$.
7. Evaluate (i) $\lim_{x\to 0}[\frac{\sin x}{x}]$ (ii) $\lim_{x\to 0} \frac{1}{1+e^{\frac{1}{x}}}$.
8. Prove that the function $f(x) = \frac{1}{x}$, $x \in (0,1)$ is not uniformly continuous on $(0,1)$.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2019-20

Sub: MATHEMATICS, Course – C6

Full Marks: 10

Answer any five questions: **Time: 30 m.**

(2 × 5 = 10)

1. Let G be a commutative group. Prove that the set H = $\{a \in G: o(a)\ divides\ 15\}$ is a subgroup of G.
2. Find the elements of order 5 in Z_{10}.
3. Prove that nth roots of unity form a cyclic group under multiplication.
4. Prove that a group of prime order is cyclic.
5. Let G be a group and $a \in G$ such that $o(a) = n$ & $a^m = e$ for some $m \in \mathbb{N}$. Prove that $n|m$.
6. Let G be a group and $Z(G) = \{x \in G: gx = xg \ \forall\ g \in G\}$. Prove that $Z(G)$ is a subgroup of G.
7. Prove that in a group G $a^2 = e \ \forall\ a \in G$. Prove that G is Abelian.
8. Prove that $(\mathbb{Z},*)$ is a group where $*$ is defined by $a * b = a + b + 1 \ \forall\ a, b \in \mathbb{Z}$.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc. (Honours) Sem. - III, 1st INTERNAL ASSESSMENT, 2019-20

Sub: MATHEMATICS, Paper- C 7 T

Full Marks: 10 **Time**: 30 m.

Answer any five of the following questions: **(5× 2 = 10)**

01. Define the significant digits. Determine the number of significant digits of the following number

$$x = 0.00265970023$$

02. Explain the **Rounding – off Error**. Round – off the following number up to 4 places of decimal

$$x = 0.00275698$$

03. Determine the relative error in computation of $x - y$ for $x = 9.05$ and $y = 6.56$ have absolute errors $\Delta x = 0.001$ and $\Delta y = 0.003$ respectively.

04. Define the operators Δ and ∇. Prove that $\Delta.\nabla = \Delta - \nabla$.

05. Estimate the missing term in the following table –

x	0	1	2	3	4	5
$f(x)$	1	3	9	?	81	243

06. State and verify the "**Fundamental Theorem of Difference Calculus**". Also derive the relation between the 1st order difference operator Δ and $D = \frac{d}{dx}$ of differential calculus.

07. Define the Shift operator. Prove that $\nabla = 1 - E^{-1}$.

08. Prove that $\Delta^n x^{(n)} = n!\, h^n$, h is the constant step length.

09. Explain the convergence criterion of the **Method of Fixed Point Iteration** for numerical approximation of the solution of the non – linear or transcendental equation.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2019-20

Sub: MATHEMATICS, Course – SEC 1

Full Marks: 5 **Time: 20 m.**

Answer any two questions: **$(2 \times 2.5 = 5)$**

1. Assuming that p *and* q are false and r *and* s are true propositions find the truth value of the proposition$((p \wedge \bar{q}) \rightarrow (q \rightarrow r) \rightarrow (s \vee \bar{q})$.

2. Examine whether the pair of propositions is logically equivalent or not
   $(p \rightarrow q) \rightarrow r$ *and* $p \rightarrow (q \rightarrow r)$.

3. Determine the truth value of the following statement where domain of discourse is the set of all real numbers. Justify your answer.
   " *for every* x , *for every* y , *if* $x < y$ *then* $x^2 < y^2$.

4. Show that $p \rightarrow q$ *and* $\bar{q} \rightarrow \bar{p}$ are logically equivalent.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – V , INTERNAL ASSESSMENT-1st , 2019-20

Sub: MATHEMATICS, Course – C11

Time: 30 m.

Full Marks: 10 **(2 × 5 = 10)**

Answer any five questions:

1. If ω be the angular velocity of a planet at the nearer end of the major axis, prove that its period is $\frac{2\pi}{\omega}\sqrt{\frac{(1+e)}{(1-e)^3}}$.
2. Write down Kepler's 2nd law on planetary motion & deduce the expression for the periodic time of a planet.
3. Prove that $h = pv$ where h, p, v are the standard notations.
4. A particle describes the parabola $p^2 = ar$ under a force which is always directed towards its focus. Find the law of force.
5. Form PDE by eliminating the function from $z = e^{ax+by} f(ax - by)$.
6. Find the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.
7. Find the complete integral of $zpq = p + q$.
8. Prove that along every characteristic strip of the PDE $f(x, y, z, p, q) = 0$ the function f is constant.

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DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – C12

Full Marks: 10
Answer any five questions: **Time: 30 m.**
$(2 \times 5 = 10)$

1. Prove that the mapping $f: U(16) \to U(16)$ $defined\ by$ $f(x) = x^3$ is an Automorphism.
2. Prove that a group G is Abelian iff $G^{/} = \{e_G\}$.
3. Prove that Z(G) is a characteristic subgroup of G.
4. Find the order of Inn(G) where G = S_3 .
5. In $\mathbb{Z}_{30} \times \mathbb{Z}_{60}$ find two subgroups of order 12.
6. Find the number of non-isomorphic Abelian group of order 360.
7. Find the order of $(10,15,24)$ in $\mathbb{Z}_{12} \times \mathbb{Z}_{30} \times \mathbb{Z}_{40}$.
8. Find all Abelian groups of order p^3q^2, where p, q are distinct primes.

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DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – V , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – DSE1

Time: 30 m.

Full Marks: 10 **$(2 \times 5 = 10)$**

Answer any five questions:

1. A manufacturer makes red and blue pens. A red pen takes twice as much time as to make a blue pen. If the manufacturer makes only blue pens, 500 can be made in a day. A red pen sells for Rs 8/- and at most 150 can be sold in a day. A blue pen sells for Rs 5/- and at most 250 can be sold in a day. The manufacturer desires to maximize his profit. Formulate the problem as linear programming problem.
2. Define convex set with an example.
3. Prove that a hyper plane is a convex set.
4. Prove that intersection of any number of convex sets is also a convex set.
5. Find the extreme points of the convex set determined by the following system of equations
   $$2x + 3y \leq 6 \ ; \ x + y \geq 1 \ , \ x, y \geq 0.$$
6. Show that the set $X = \{(x, y): x \leq 5, y \geq 3\}$ is a convex set.
7. Find the extreme points of the feasible space of the following LPP by graphical method.
   Maximize $Z = x_1 + 2x_2$
   Subject to $x_1 + x_2 \leq 2 \ ; \ x_1 - x_2 \geq 1 \, , x_1, x_2 \geq 0.$
8. Find the maximum value of the objective function of the LPP by graphical method
   Maximize $Z = 10x_1 + 15x_2$
   Subject to $x_1 + x_2 \geq 2 \ ; \ 3x_1 + 2x_2 \leq 6 \, , x_1, x_2 \geq 0.$

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc. (Honours) Sem. - V, 1st INTERNAL ASSESSMENT, 2019-20

Sub: MATHEMATICS, Paper - DSE2

Full Marks: 10 **Time**: 30 m.

Answer any five of the following questions: (5× 2 = 10)

01. Prove the following identity –
$\lim_{x\to a+0} F(x) = F(a)$, where $F(x)$ is the distribution function of a random variable X connected to a random experiment R.E.

02. Let X be continuous random variable with probability density function $f(x)$ is given by –

$$f(x) = \begin{cases} kx, 0 \le x \le 1 \\ k, 1 \le x \le 2 \\ -kx + 3k, 2 \le x \le 3 \\ 0, x > 3 \end{cases}$$

. Determine k and also find the distribution function $F(x)$ of the random variable.

03. Five balls are drawn from an urn containing 3 white and 7 black balls. Find the probability distribution of the number of white balls drawn without replacement.

04. If X has a gamma distribution with parameter l, find the distribution of the random variable $Y = \frac{1}{2}X^2$.

05. X is a continuous random variable with probability density function $f(x)$ given by -

$$f(x) = \begin{cases} \frac{2}{x^2} \text{ if } x \ge 1 \\ 0, \text{ if } x < 1 \end{cases}$$

. Show that $E(X)$ exists but $E(X^2)$ does not exists.

06. Prove that the standard deviation is dependent on the unit of measurement but independent of the choice of origin of measurement.

07. If the probability density function of a random variable X is given by $f(x) = Ke^{-(x^2+2x+3)}, -\infty < x < \infty$, find the value of K and also the expectation of the random variable.

08. Find the expected value of the product of the number on n dice tossed together.

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DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – II , INTERNAL ASSESSMENT-2nd , 2018-19

Sub: MATHEMATICS, Course – C3

Full Marks: 10 **Time: 30 m.**

Answer any five questions: **$(2 \times 5 = 10)$**

1. Let A and B be subsets of R of which A is closed and B is compact. Prove that $A \cap B$ is also compact subset of R.
   [**Compact Set in R**: A set is said to be a compact if every open cover of the set has a finite sub-cover]
2. Prove that the set of all circles in the plane having rational radii and centres with rational coordinates is an enumerable set.
3. If a boundary point of a set S is not a point of S prove that it is limit point of the set.
4. Show that for any fixed value of x, the series $\sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$ is convergent.
5. When is a series said to converge conditionally?
6. Prove that a bounded sequence $\{u_n\}$ is convergent iff $\overline{lim} u_n = \underline{lim} u_n$.
7. Prove that $\lim_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n} = \frac{1}{e}$.
8. If $\{u_n\}$ be a Cauchy sequence in $\mathbb{R}$ having a subsequence converging to a real number l, Prove that $\lim_{n\to\infty} u_n = l$.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – II , INTERNAL ASSESSMENT-2nd , 2018-19

Sub: MATHEMATICS, Course – C4

…l Marks: 10

Answer any five questions:

Time: 30 m.

($2 \times 5 = 10$)

1. Prove that $\sin x, \sin 2x, \sin 3x$ are linearly independent on $[0,2\pi]$.
2. Linear combinations of solutions of an ordinary differential equation are solutions if the differential equation is
   (a) Linear non – Homogeneous
   (b) Linear Homogeneous
   (c) Non – Linear Homogeneous
   (d) Non – Linear non – Homogeneous
3. Prove that $x = 1$ is a regular Singular point of the following differential equation
   $x^3(x^2-1)\frac{d^2y}{dx^2} + 2x^4\frac{dy}{dx} + 4y = 0.$
4. Find the Singular points of the differential equation $(x^2-9)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = 0.$
5. Solve: $\frac{dx}{1} = \frac{dy}{2} = \frac{dz}{5z+\tan(y-2x)}$
6. Solve: $\frac{dx}{dt} = 4x - 2y$, $\frac{dy}{dt} = x + y.$
7. Solve: $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{z}$
8. Show that the points with position vectors $2\vec{i} - 3\vec{j} + \vec{k}, 3\vec{i} + 2\vec{j} - 5\vec{k}$, $\vec{i} + 4\vec{j} + 7\vec{k}, 2\vec{i} + \vec{j} + \vec{k}$ are coplanar.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE

B.Sc (Honours) Sem - IV, 2nd INTERNAL ASSESSMENT, 2018-19
Sub: MATHEMATICS, Paper- CC 8

Marks: 10 **Time**: 30 m.

Answer the following questions: **(2 × 5 = 10)**

01. Let $f:[a,b] \to R$ be defined as $f(x) = \begin{cases} a_{n+1}, \text{ if } x = n \in [0,2019] \cap Z \\ 0, \text{ otherwise} \end{cases}$. Prove that the function f is Riemann Integrable and evaluate $\int_0^{2019} f$.
[*Symbols have their usual meaning*]

02. Let $f(x) = [x], x \in [1,3]; \varphi(x) = \begin{cases} x, x \in [1,2] \\ 2x-2, x \in]2,3] \end{cases}$. Show that the given function f is Riemann Integrable function and without evaluating the integral show that $\int_1^3 f = \varphi(3) - \varphi(1)$.

03. Let $f,g:[a,b] \to R$ be both Riemann Integrable functions on $[a,b]$. Prove that $max(f,g):[a,b] \to R$ is also Riemann Integrable function.

04. For each $n \in N$, let $f_n(x) = x - \frac{1}{n}, g_n(x) = x + \frac{2}{n}, 0 \le x < \infty$. Show that the given sequences are uniformly convergent on $[0,\infty[$. Determine the nature of the sequence $\{f_n g_n\}$.

05. Prove that the uniform limit of a sequence of continuous functions is continuous on the same domain of definition.

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