বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2020

(Under CBCS Pattern)

Semester - V

Subject: MATHEMATICS

Paper: SEC3T

Full Marks : 40

Time : 2 Hours

Candiates are required to give their answer in their own words as far as practicable.

The figures in the margin indicate full marks.

NUMBER THEORY

Answer any ***two*** questions. 2×20=40

1. Answer any ***five*** questions : 5×4=20

(i) If $a, b \in \mathbb{Z}$, then prove that $a|b$ always implies $a|bm$ for any integer *m*.

(ii) Prove that, for $n > 3$, the integer n, $n+2$, $n+4$ cannot be all primes.

(iii) Let us consider two integers 287 and 271. Determine whether they are primes or not.

(iv) Show that for any two integers a and b, $(a+b, [a,b]) = (a,b)$.

(v) Find the remainder when 7^{30} is divided by 4.

(vi) Find $\tau(360)$ and $\sigma(360)$.

(vii) Find the number of zeros in 50!.

(viii) Prove that $1!+2!+3!+...+1000! \equiv 3 \pmod{15}$.

2. (a) Find the $\gcd(120, 275)$ and express it in the form $120u + 275v$ where u, v are integers. 10

(b) Find the remainder when $1^5 + 2^5 + 3^5 + ... + 100^5$ is divisible by 4. 10

3. (a) Find all solutions of the Diophantine equation $108x + 45y = 81$.

(b) If n is a positive integer and p is a prime, then prove that the exponent of the highest power of p that divides $n!$ Is $\sum_{k=1}^{\infty}\left[\frac{n}{p^k}\right]$, where $[x]$ is the greatest integer function.

4. (a) State and prove Mobius inversion formula. 10

(b) If $2^n - 1$ be prime, prove that n is prime. 10

BIO-MATHEMATICS

Answer any ***two*** questions. 2×20=40

1. (a) Write a short note on Malthus model. 5

   (b) Discuss logistic law of growth and Gompertz growth law in population Biology. 5+5

   (c) What is Allec effect ? 5

2. For the system of ordinary differential equation $\left.\begin{aligned}\frac{dx}{dt} &= x \\ \frac{dy}{dt} &= -x + 2y\end{aligned}\right\}.$

   Find the critical point of the system. Discuss the type of stability of the critical point. Write down the general solution of the system. Draw the graph of the trajectories. 2+7+7+4

3. (a) Define Class of susceptible, Class of infectives and Class of removals in Epidemic models. 6

   (b) Some disease are spread largely by carriers, individuals who can transmit the disease but who exhibit no overt symptoms. Let S and I, respectively denote the proportion of susceptible and carriers in the population. Suppose that carriers are identified and removed from the population at a rate β so that $\frac{dI}{dt} = -\beta I$.

   Also, suppose that the disease spreads at a rate proportional to the product of S and I, this is $\frac{ds}{dt} = -\alpha SI$.

   (i) Determine the proportion of carriers at any point t.

   (ii) Using the above result find susceptible at time t, where initially $S(0) = S_0$.

   (iii) Find the proportion of the population that escape the epidemic. 5+5+4

4. A prey-predator model satisfies the differential equation

$$\frac{dx}{dt} = \alpha x - \beta xy \; ; \; \alpha > 0, \; \beta > 0$$

$$\frac{dy}{dt} = \in \beta xy - \delta y \; ; \; \in > 0, \; \delta > 0$$

with $x(0) = x_0$, $y(0) = y_0$ and $x(t)$, $y(t)$ represent the population of prey and predator at time *t*, respectively.

(i) Discuss the stability of the equilibrium poins.

(ii) Explain the limitations of the above prey-predator model. 12+8

MATHEMATICAL MODELING

Answer any ***two*** questions. 2×20=40

1. (i) If a particle moves in a straight line in such a manner that its acceleration is always proportional to its distance from the origin and is always directed towards the origin, then prove that the dynamical equation is $v\frac{dv}{dt} = -\mu x$. 7

   (ii) A particle falls under gravity (*g*) in a medium in which the resistance is proportional to the velocity (*v*). Then prove that the velocity equation is given by $v = V\left(1-e^{-kt}\right)$. 6

   (iii) Suppose that a body moves through a resisting medium with resistance proportional to its velocity *v*, so that $\frac{dv}{dt} = -kv$. Then show that its velocity and position at time *t* are given by $v(t) = v_0 e^{-kt}$ and $x(t) = x_0 + \frac{v_0\left(1-e^{-kt}\right)}{k}$. 7

2. (i) A 32-lb weight is attached to the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby streatching the spring 2 ft. The weight is then pulled down 6 in. below its equilibrium position and released at $t = 0$. No external forces are present; but the resistance of the medium in pounds is numerically equal to $4\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second. Determine the resulting motion of the weight on the spring. 10

   (ii) A circuit has in series an electromotive force given by $E = 100\sin 40t$ V, a resistor of 10Ω and an inductor of 0.5 H. If the initial current is 0, find the current at time $t > 0$. 10

3. (i) A 6-lb weight is hung on the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 4 in. Then beginning at $t = 0$ an external force given by $F(t) = 27\sin 4t - 3\cos 4t$ is applied to the system. If the medium offers a resistance in pounds numerically equal to three times the instantaneous velocity, measured in feet per second, find the displacement as a function of the time. 10

(ii) An 8-lb weight is placed upon the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 6 in. The weight is then pulled down 3 in. below its equilibrium position and released at $t=0$ with an initial velocity of 1 ft/sec, directed downward. Neglecting the resistance of the medium and assuming that no external forces are present, determine the amplitude, period and frequency of resulting motion. 10

4. (i) Find the deflection $y(x,t)$ of the vibrating string (length = π, and $c^2=1$) corresponding to zero initial velocity and initial deflection $f(x)=k(\sin x-\sin 2x)$. 10

(ii) Solve the system of equations : $\frac{dx}{dt}-7x+y=0$, $\frac{dy}{dt}-2x-5y=0$. 10