বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2020

(Under CBCS Pattern)

Semester - I

Subject: MATHEMATICS

Paper: DSC 1A/2A/3A-T

(Differential Calculus)

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any ***three*** from the following questions : 3×20

1. (a) If $f(x)=\begin{cases}\dfrac{a\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 7 & \text{if } x=\dfrac{\pi}{2}\end{cases}$ for what values of 'a' $\lim\limits_{x\to\frac{\pi}{2}} f(x)=f\left(\dfrac{\pi}{2}\right)$?

   (b) If $y=x^{n-1}\log x$, then show that $y_n=\dfrac{(n-1)!}{x}$.

   (c) If a and b are distinct real numbers, show that there exists a real number c between a and b such that $a^2+ab+b^2=3c^2$.

(d) Find the radius of curvarture of the curvey $y = xe^{-x}$ at the point where y is maximum.

(e) If $f(x+y) = f(x).f(y)$, $f'(0) = 3$, $f(5) = 2$, find $f'(5)$. 4×5=20

2. (a) If $f(x) = \tan x$, then show that $f^n(0) - n_{C_2} f^{n-2}(0) + n_{C_4} f^{n-4}(0) - \ldots = \sin\frac{n\pi}{2}$

(b) If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$ 10+10

3. (a) If $lx + my = 1$ is a normal of the parabola $y^2 = 4ax$, then show that $al^3 + 2alm^2 = m^2$.

(b) Find the value of a, so that $\lim_{x\to 0}\frac{a\sin x - \sin 2x}{\tan^3 x}$ exists finitely and find the limit. 10+10

4. (a) A function $f:[0,1]$ is continuous on $[0,1]$. Prove that there exists a point c in $[0,1]$ such that $f(c) = c$.

(b) Using mean value theorem show that $0 < \frac{1}{x}\log\left(\frac{e^x - 1}{x}\right) < 1$, for $x > 0$. 10+10

5. (a) If p_1, p_2 be the radii of curvarture at the extremities of any chord of the cardioid $r = a(1+\cos\theta)$, which passes through the pole, then prove that $p_1^2 + p_2^2 = \frac{16a^2}{9}$.

(b) Find the infinite series expansion of the function $\sin x$, $x \in \mathbb{R}$. 10+10

6. (a) Show that $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 \quad , & (x,y) = (0,0) \end{cases}$ has partical derivatives but is not continuous at (0, 0).

(b) If the normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ makes an angel ϕ with the axis of x, show that its equation is $y\cos\phi - x\sin\phi = a\cos 2\phi$. 10+10