বিদ্যাসাগর বিশ্ববিদ্যালয়

# VIDYASAGAR UNIVERSITY

## Question Paper

## B.Sc. Honours Examinations 2020

**(Under CBCS Pattern)**

## Semester - I

## Subject: MATHEMATICS

**Paper: C 2-T**

**Full Marks : 60**

**Time : 3 Hours**

*Candidates are required to give their answers in their own words as far as practicable.*
*The figures in the margin indicate full marks.*

Answer any ***three*** from the following questions : 3×20

1. (a) If $A$ and $P$ be both $n \times n$ matrices and $P$ be non singular, then $A$ and $P^{-1}AP$ have the same eigen values. 2

(b) If $a$ is prime to $b$, prove that $a+b$ is prime to $ab$. 2

(c) $Z$ is a complex number satisfying the condition $\left|z-\frac{3}{z}\right|=2$. Find the greatest and the least value of $|z|$. 2

(d) $A$ and $B$ are real orthogonal matrices of the same order and $|A|+|B|=0$. Show that $A+B$ is a singular matrix. 2

(e) In n be a positive integer and $(7+2i)^n = a+ib$, then prove that $a^2+b^2=(53)^n$. Hence express $(53)^2$ as the sum of two squares. 2

(f) Examine if the set $S=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2=z^2\}$ is a subspace of $\mathbb{R}^3$. 2

(g) If $2^n-1$ be a prime, prove that $n$ is a prime. 2

(h) If $n$ be a positive integer greater than 2, then prove that $(n!)^2 > n^n$. 2

(i) If the roots $\alpha,\beta,\gamma$ of the equation $x^3+qx+r=0$ be in A.P then show that the rank of the matrix $\begin{pmatrix}\alpha & \beta & \gamma\\ \beta & \gamma & \alpha\\ \gamma & \alpha & \beta\end{pmatrix}$ is 2. 2

(j) Define eigen value of a matrix of order $n$. If $\lambda$ be an eigen value of an $n\times n$ idempotent matrix A, then prove that $\lambda$ is either 1 or 0. 2

2. (a) Find eigen values and a basis of each eigen space for the operator $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(2x+y,\ y-z,\ 2y+4z)$. 6

(b) Find the roots of $z^n=(z+1)^n$, where $n$ is a positive integer, and show that the points which represent them in the Argand diagram are collinear. 6

(c) If the roots of the equation $a_0x^n+na_1x^{n-1}+\frac{n(n-1)}{2!}a_2x^{n-2}+\ldots+a_n=0$ be in A.P., show that they can be determined from the expression

$$-\frac{a_1}{a_0}\pm\frac{r}{a_0}\sqrt{\left[\frac{3\left(a_1^2-a_0a_2\right)}{n+1}\right]}$$

by giving r the values 1, 3, 5, ..., $n-1$ when $n$ is even and all the values 0, 2, 4, ..., $n-1$ when $n$ is odd. 8

3. (a) Prove that interchange of two rows does not alter the rank of a matrix. 5

(b) Prove that the product of any $m$ consecutive integers is divisible by $m$. 5

(c) For what integral values of $m, x^2+x+1$ is a factor of $x^{2m}+x^m+1$ ? 6

(d) If $\alpha$ be a root of the equation $x^3-3x-1=0$, prove that the other roots are $2-\alpha^2, \alpha^2-\alpha-2$. 2

(e) If $i^{\alpha+i\beta}=\alpha+i\beta$ then prove that $\alpha^2+\beta^2=e^{-(4n+1)\pi\beta}$. 2

4. (a) Solve completely the equation $x^4-5x^3+11x^2-13x+6=0$ using the fact that two of its roots $\alpha$ and $\beta$ are connected by the relation $3\alpha+2\beta=7$. 8

(b) If n be positive integer, prove that $\frac{1}{\sqrt{4n+1}}<\frac{3.7.11...(4n-1)}{5.9.13...(4n+1)}<\sqrt{\frac{3}{4n+3}}$ 6

(c) Find the maximum value of $(x+2)^5(7-x)^4$ when $-2<x<7$. 2

(d) Prove that the vector space P of all real polynomials is infinite dimensional. 2

(e) Define a basis of a vector space. Prove that the rank of a vector space is unique. 2

5. (a) Find for what values of $a$ and $b$ the following system of equations has (i) a unique solution (ii) no solution (iii) infinite number of solutions over the field of rational numbers $x_1+4x_2+2x_3=1, 2x_1+7x_2+5x_3=2b, 4x_1+ax_2+10x_3=2b+1$. 8

(b) Prove that $V$ is the vector space of polynomials in $x$ of degree $\leq n$ over $\mathbb{R}$. Show that the set $S=\{1, x, x^2, ..., x^n\}$ is $a$ basis of $V$. 6

(c) Prove that $x^8+y^8=\prod\left(x^2-2xy\cos\frac{r\pi}{8}+y^2\right)$, $r=1,3,5,7$. 6

6. (a) Prove that for any two integers $a$ and $b$, $a\equiv b\,(\text{mod } m)$ iff $a$ an $b$ leave the same remainder when divided by $m$. 6

(b) If $\alpha, \beta, \gamma$ b the roots of $x^3-qx+r=0$, find the equation whose roots are

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}-\frac{1}{\gamma^2}, \frac{1}{\beta^2}+\frac{1}{\gamma^2}-\frac{1}{\alpha^2}, \frac{1}{\gamma^2}+\frac{1}{\alpha^2}-\frac{1}{\beta^2}$$

and hence calculate the value of

$$\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}-\frac{1}{\gamma^2}\right)\left(\frac{1}{\beta^2}+\frac{1}{\gamma^2}-\frac{1}{\alpha^2}\right)\left(\frac{1}{\gamma^2}+\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right)$$ 8

(c) If $a_1, a_2, ..., a_n; b_1, b_2, ..., b_n$ be all real numbers, then show that

$\left(a_1^2+a_2^2+....+a_n^2\right)\left(b_1^2+b_2^2+...+b_n^2\right)>\left(a_1b_1+a_2b_2+...+a_nb_n\right)$, when $\left(a_1,\ a_2,\ ...,\ a_n\right)$

and $\left(b_1, b_2, ..., b_n\right)$ are not proportional. 6