Total Pages - 20 **UG/5th Sem/Math(H)/T/19**

2019

B.Sc. (Honours)

5th Semester Examination

MATHEMATICS

Paper - DSE-2T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

(PROBABILITY AND STATISTICS)

Unit - I

(Probability and Distribution)

1. Answer any *three* questions : $3\times2=6$

(a) A box contains 'a' white and 'b' black balls : c balls are drown. Find the expectation of the number of white balls drawn.

[Turn Over]

5/44-2400

(b) Show that

$$P(\overline{A} \cap \overline{B}) = P(\overline{A}) - P(B) + P(A \cap B)$$

(c) If X be continuous random variable, prove that $P(X = a) = 0$ for every real number 'a'.

(d) Let X be a continuous random variable having distribution function $F(x)$. Show that $Y = F(x)$ has uniform distribution over (0, 1).

(e) The probability density function of a random variable X is symmetric about the origin. Prove that X and $-X$ have the same distribution.

2. Answer any *two* questions : $5 \times 2 = 10$

(a) A continuous random variable X has the probability density function $f(x) = ae^{-ax}$, $0 < x < \infty$ (a is positive constant). Obtain the moment generating function of X and hence find $E(X^n)$. 5

(b) In the equation $x^2 + 2x - Q = 0$, Q is a random variate uniformly distributed over the interval (0, 2). Find the distribution of the larger roots.

(c) A point P is chosen at random on a line segment AB of length 2 cm. Calculate the expected values of $AP \cdot PB$ and $|AP - PB|$. 3+2

Unit - II

(Joint Distribution)

3. Answer any *two* questions : 2×2=4

(a) If $f(x, y)$ is a non-negative function satisfying

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\,dxdy = 1,$$ then show that $f(x, y)$ is a density function of two-dimensional random variable X and Y.

(b) If X is a $\gamma\left(\frac{n}{2}\right)$ variate, then show that $Y = 2X$ has a χ^2- distribution with n degrees of freedom.

[Turn Over]

(c) Define correlation co-efficient between the random variable X and Y. What is significance of that is zero ?

4. Answer any *one* question : 10×1=10

(a) (i) The joint density function of the random variates X, Y is given by

$$f(x, y) = 2, \ (0 < x < 1, \ 0 < y < x).$$

Find the marginal and conditional density function and compute

$$P\left(\frac{1}{4} < X < \frac{3}{4} \middle/ Y = \frac{1}{2}\right)$$

(ii) If (X, Y) has the normal distribution in two-dimensions with zero means, unit variances and correlation co-efficient ρ, then prove that the expectation of the greater of X and Y is $\sqrt{(1-\rho)/\pi}$.

5

(b) (i) For a bivariate random variable (X, Y), define regression curves. For a bivariate normal distribution, prove that regression curves are identical with regression lines. 1+4

(ii) Let X and Y be independent poisson variates with parameter λ and μ. Show that the conditional distribution of X given that $X+Y=n$ is binomial whose n is positive integer. 5

Unit - III

(Convergence in Probability)

5. Answer any *two* questions : 2×2=4

(a) State Tchebycheff's inequality and give the physical significance of it.

(b) Show that poisson distribution as a limit of the binomial distribution.

(c) If X is a poisson - 3 random variable, then show that $P(|X-3|<1)=\dfrac{9}{2e^3}$.

6. Answer any *one* question : 5×1=5

(a) Let $\{X_i\}$ be a sequence of independent random variables such that for each $E(X_i)=m_i$, $\operatorname{var}(X_i)=\sigma_i^2 \le \sigma^2 < \infty$. Use Tchebycheff's inequality to show that

$$\sum_{i=1}^{n}\frac{X_i}{n}-\sum_{i=1}^{n}\frac{m_i}{n} \xrightarrow[\text{in } p]{} 0 \text{ as } n \to \infty$$

(b) A random variable X has the probability density function given by

$$f(x)=\begin{cases} 12x^2(1-x), & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

Compute $P(|X-m_x| \ge 2\sigma_x)$ and compare it with the limit given by Tchebycheff's inequality where m_x and σ_x are mean and standard deviation of X.

Unit - IV

(Statistics)

7. Answer any *three* questions : $2\times3=6$

   (a) What do you mean by type-I and type-II error in testing of hypothesis ?

   (b) Explain the terms : Statistical regularity, stochastically impossible event.

   (c) Find the sampling distribution of the sample mean for the normal population. 2

   (d) What do you mean by confidence interval in connection to interval estimation of a statistic ?

   (e) State Neyman-Pearson theorem in connection with best critical region.

8. Answer any *one* question : $5\times1=5$

   (a) Obtain an unbiased as well as a consistent estimate of the population variance σ^2. Also show that

[Turn Over]

$$E\left\{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2\right\}=\sigma^2,\ n>1$$

(b) A die was thrown 102 times and the frequencies of the different faces were observed to be the following :

face	: 1	2	3	4	5	6
frequency	: 16	17	19	18	9	23

Test at significance level 0.10, whether the die is honest, given that $\chi^2_{0.10}(5)=9.24$.

9. Answer any *one* question : 10×1=10

(a) (i) Find the maximum likelihood estimate for θ when the probability density function is defined as —

$$f(t)=\frac{1}{\theta}\,e^{-\frac{t}{\theta}},\ 0<t<\infty. \qquad 5$$

(ii) A drug is given to 10 patients and the increments in the blood pressure where recorded to be 3, 6, –2, 4, –3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on change of blood pressure ? (Given $P(t > 2.622) = 0.025$ for 9 degrees of freedom. 5

(b) Find out a $100(1-\alpha)\%$ confidence interval for the mean of a normal (m, σ) population on the basis of a sample of size n drawn from the population. Hence find the 95% confidence limits for the mean score of the population of 10 years old childrens in a psychological test is known to have a standard deviation 5.2 if a random sample of size 20. Show a mean of 16.9. Assuming that the population is normal.

[Given $P(|U| < 1.96 = 0.95)$] 6+4

(BOOLEAN ALGEBRA AND AUTOMATA)

Group - A

1. Answer any *ten* questions out of 15 questions : $2\times10=20$

(a) Draw $F = AB'C + C'D$.

(b) If $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 8\}$,

$C = \{2, 5, 7, 8\}$ verify that

$$A - (B \cup C) = (A - B) \cap (A - C).$$

(c) Let A and B be two finite sets such that $n(A - B) = 30$, $n(A \cup B) = 180$,

$n(A \cap B) = 60$, find $n(B)$.

(d) Write two important characteristics of digital IC.

(e) Prove $(x + y)(x + z) = x + yz$

(f) How does DFA differ with NFA ?

(g) Show that the set of integers $\mathbb{Z}$ is countable.

(h) Construct the truth table for the compound proposition : $(p \vee q) \rightarrow (p \oplus q)$.

(i) Using 1's complement method, subtract $(1011.01)_2$ from $(11001.101)_2$.

(j) Implement XOR using minimum number of universal gates.

(k) Find the type of the following production :

$aA \rightarrow abC$

(l) Find the regular expression over $\{a, b\}$ where the strings either start or end with '*ab*'.

(m) State Arden's theorem regarding regular expression.

(n) Give an example of an ambiguous grammar.

(o) What do you mean by recursive language ?

[Turn Over]

Group - B

2. Answer any *four* questions : 5×4=20

(a) Prove that if L_1 and L_2 are recursively enumerable languages then $L_1 \cup L_2$ is also recursively enumerable.

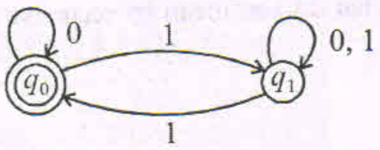
(b) Prove that set of regular language is closed under complements.

(c) If $R = \{(1,2), (2,3), (2,4)\}$ be a relation in $\{1,2,3,4\}$, find R^+.

(d) Minimize the following expression using Karnaugh maps method.

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

(e) Construct a deterministic automaton equivalent to the NFA given by :



(f) Design a PDA that recognizes the language

$$L = \left\{ a^n \, b^n \, \middle| \, n \geq 1 \middle| \right\}$$

Group - C

3. Answer any *two* questions. $10 \times 2 = 20$

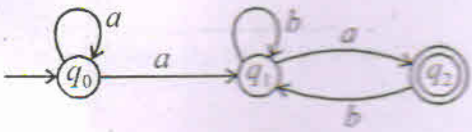
(a) Boolean function F defined on 3 input variables X, Y, Z is 1 if and only if number of 1 input is odd. Draw the truth table for the above function and express it in canonical SOP and POS form.

(b) Using pumping lemma, show that the language

$$L = \left\{ a^i b^j c^k \middle/ i = j = k \text{ and } i, j, k \geq 1 \right\}$$

is not context free.

(c) (i) Find the regular expression accepted by the following FA :



(ii) Construct a DFA accepting all strings w over $\{0, 1\}$ such that the number of 0's in w is divisible by 3. 6+4

(d) (i) Prove that $L = \{a^p \mid p \text{ is prime}\}$ is not regular.

(ii) Minimize the FA :

State	input = a	input = b
$\rightarrow q_1$	q_2	q_6
q_2	q_7	q_3
q_3	q_1	q_3
q_4	q_3	q_7
q_5	q_8	q_6
q_6	q_3	q_7
q_7	q_7	q_5
q_8	q_7	q_3

6+4

(PORTFOLIO OPTIMIZATION)

1. Answer any *ten* of the following : $2 \times 10 = 20$

   (i) What is systematic risk ?

   (ii) Explain the term 'risk' in the case of a portfolio.

   (iii) What do you mean by beta of 1.5 for a security ?

   (iv) What is a mutual fund ?

   (v) Write down the formula of portfolio risk in the case of a three-security portfolio.

   (vi) What is an index fund ?

   (vii) Write down the name of two highly risky and two low risky securities.

   (viii) Write down two objectives of investment.

   (ix) Explain entry load and exit load.

   (x) Is speculation the same às investment ?

(xi) What is the role of diversification ?

(xii) What is the difference between correlation and covariance between securities ?

(xiii) What is 'risk' in the case of a security ?

(xiv) Explain the function of capital market ?

(xv) What is holding period rate of return ? Give an example.

2. Answer any *four* of the following : 5×4=20

(i) What are the advantages of investing in a mutual fund ?

(ii) How does the security market line help in identifying under-priced and over-priced securities ?

(iii) Write down the meaning and importance of NAV. How is it computed ?

(iv) Write a short note on the capital market line.

(v) Explain Jensen's measure for evaluating portfolios.

(vi) Discuss the importance of capital asset pricing model in investment decisions.

3. Answer any *two* of the following : 10×2=20

(a) (i) Mrs. Sangita has approached you to guide her relating to her investment decision. She gives you the following information relating to three mutual funds that she is considering for her investment :

Mutual Fund	Average return	Beta	Standard deviation
Uproar	15%	1.25	17%
Jovial	16.5%	1.10	14.8%
Happy	18.5%	1.50	15.6%
Nifty	13.8%	1.00	11.8%

Assuming the risk-free rate of return to be 5.9%, you are required to suggest the best investment for her based on Treynor's measure. 6

(ii) You are given the following data relating to a portfolio having two securities M and N, the details of which are given below :

Particulars	Security M	Security N
Return (%)	14.2	15.3
Standard deviation (%)	11.5	12.8
Covariance MN	147.20	
Investment ratio	2 : 3	

The following are to be determined :

* Portfolio risk

* Investment ratio required to reduce the portfolio risk to zero. 4

(b) (i) What do you mean by efficient frontier ? Discuss. 5

(ii) There are two securities U and V. You are given the following information —

State of the economy	Probability	Return (%)	
		Security U	Security V
Good	0.50	18	16
Moderate	0.30	13	14
Gloomy	0.20	10	11

You are required to compute the following :

1. The correlation between securities *U* and *V*.

2. The portfolio expected return and risk, assuming that in the 2-security portfolio *UV*, investment in *U* and *V* to made in the ratio of 1 : 4. 5

(c) (i) There is a portfolio having three securities E, F and G. The beta of the individual securities is 2.1, 1.8 and 1.5 respectively. If the rattio of investment in these three secrities is 1:2:3, calculate the beta of the portfolio. 5

(ii) Mention the differencesw beween capital market and money market. 5