2019

B.Sc.

3rd Semister Examination

MATHEMATICS (Honours)

Paper - C 7-T

Full Marks : 40 Time : 2 Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practiable.
Illustrate the answers wherever necessary.

Unit - I

1. Answer any *two* questions : 2×2=4

(a) Given $f(x, y, z) = \frac{5xy^2}{z^2}$, find the relative maximum error in the evaluation of f(x, y, z) at $x = y = z = 1$, if x, y, z have absolute error $\Delta x = \Delta y = \Delta z = 0.1$.

(b) Define the terms :

(i) Computational error, and

(ii) Relative percentage error

(c) Three approximate values of the number $\frac{1}{3}$ are given as 0.30, 0.33 and 0.34. Which of these three is the best approximation?

Unit - II

2. Answer any *one* question : $2\times1=2$

(a) Discuss the condition of convergence of Newton-Raphson method.

(b) Prove that the order of convergence of iteration method is linear.

3. Answer any *one* question : $5\times1=5$

(a) Explain the method of Iteration for computing a real root of an equation $f(x) = 0$. Let the iteration function $\phi(x)$ maps the interval [a, b] into itself and is differentiable there. Further there exists a non negative constant $k < 1$ such that $\forall x$ in [a, b], $|\phi'(x)| \leq k$ then prove that $\phi(x)$ has exactly one fixed point α on [a, b] and the sequence $\{x_n\}$ converges to α. 5

(b) (i) Show that the square root of $N = AB$ is given by $\sqrt{N} \simeq \frac{S}{4} + \frac{N}{S}$ where $S = A + B$.

(ii) Derive the expression for Secant method to find the root of an equation. 2+3

Unit - III

4. Answer any *one* question : 2×1=2

(a) What is called pivoting? Why pivoting is necessary to solve a system of equations using Gaussian ellimination method?

(b) State the difference between direct and iterative methods.

5. Answer any *one* question : 5×1=5

(a) Describe Gauss Jacobi method for solution of a system of linear equation. State the sufficient condition for convergene of this method.

(b) Solve the following equations by Gauss-Jordan elimination method :

$$x_1 + x_2 + x_3 = 3$$

$$2x_1 + 3x_2 + x_3 = 6$$

$$x_1 - x_2 - x_3 = -3$$

[Turn Over]

Unit - IV

6. Answer any *one* question : $10 \times 1 = 10$

(a) (i) Given that $f(0) = 2$, $f(1) = 4$, $f(2) = 6$, $f(3) = 10$ and 3rd difference being constant. Find $f(5)$.

(ii) Prove that a divided difference is symmetric function of its arguments. If $f(x) = x^2$ then prove that $f[x_0\ x_1\ x_2]$ is constant and all higher order difference are zero.

(iii) In a country school going children of a certain age group is given for different years as follows :

Year	1995	2000	2005	2010	2015
No. of student (in thousand)	304	329	357	387	421

Estimate the number in the year 2020.

(b) Establish Lagrange's interpolation formula. Show that the Lagrangian functions are invariant under a linear transformation. 7+3

Unit - V

7. Answer any *one* question : 2×1=2

   (a) Why does one need to use numerical method instead of analytical method or integration.

   (b) What is degree of precision. What is the degree of precision of Weddle's rule ?

8. Answer any *one* question : 5×1=5

   (a) Using power method, find the largest eigen value in magnitude and corresponding eigen vector of the matrix $A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 0 & 2 \\ 3 & 4 & 5 \end{pmatrix}$.

   (b) Derive Trapezoidal rule from general quadrature formula and discuss its geometrical significance.

Unit - VI

9. Answer any *one* question : 5×1=5

   (a) Define single step and multistep methods. Use R-K method of order 2 to approximate y when

$x = 0.1, 0.2, 0.3$ given that $\frac{dy}{dx} = y - x$, $y(0) = 2$.

(b) Write down the working rule of modified Euler's method for solving first order differential equation with initial condition. Comments on accuracy of Euler's method in solving a differential equation.