UG/1st Sem/MATH(H)/T/19

2019

B.Sc.

1st Semester Examination

MATHEMATICS (Honours)

Paper - C 1-T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.

Unit - I

1. Answer any *three* of the following questions : 3×2=6

(a) If $y = c^{ax}\cos^2 bx$, find $y_n\left(a,b>0\right)$.

(b) Find the oblique asymptotes of the curve

$$y=\frac{3x}{2}\log\left(e-\frac{1}{3x}\right)$$

[Turn Over]

(c) If $y = x^{n-1}\log x$, then prove that $y_n = \dfrac{(n-1)!}{x}$.

(d) What is reciprocal spiral? Sketch it.

(e) The parabolic path is given by

$$y = x\tan\theta - \frac{x^2}{4h\cos^2\theta}$$

what will be the asymptote of parabolic paths ?

2. Answer any *one* questions : $1\times10=10$

(a) (i) Find the evolute of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$. 5

(ii) Let $P_n = D^n\left(x^n\log x\right)$.

Prove that $P_n = nP_{n-1} + \lfloor\underline{n-1}$. Hence show that $P_n = n!\left(\log x + 1 + \dfrac{1}{2} + \ldots + \dfrac{1}{n}\right)$. 5

(b) (i) Prove that the envelope of circles whose centres lie on the rectangular hyperbola $xy = c^2$ and which pass through its centre is $\left(x^2 + y^2\right)^2 = 16c^2xy$. 5

(ii) Find the point of inflexion on the curve $\left(\theta^2 - 1\right)r = a\theta^2$. 5

Unit - II

3. Answer any *two* questions : 2×2=4

(a) If $I_n = \int_0^{\pi/2} \cos^{n-2} x \sin x \, dx, n > 2$. Prove that $2(n-1)I_n = 1 + (n-2)I_{n-1}$.

(b) Find the length of the curve

$$x = e^\theta \sin\theta \text{ and } y = e^\theta \cos\theta$$

between $\theta = 0$ to $\theta = \frac{\pi}{2}$.

(c) Find the reduction formula for

$$\int \cos^m x \sin(nx) \, dx.$$

4. Answer any *two* questions : 2×5=10

(a) Prove that the volume of the solid obtained by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the initial line is $\frac{1}{2}\pi a^3\left\{\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)-\frac{1}{3}\right\}$.

(b) If $I_{m,n} = \int_0^1 x^m (1-x)^n dx$,

where m and n are positive integers, then prove that $(m+n+1)I_{m,n} = nI_{m,n-1}$ and deduce that

$$\mathrm{I}_{m,n} = \frac{m!n!}{(m+n+1)!}.$$

(c) Evaluate the surface area of the solid generated by revolving the cycloid

$x = a(\theta - \sin\theta), y = a(1-\cos\theta)$ about the line $y = 0$.

Unit - III

5. Answer any *three* questions : 3×2=6

(a) Find the centre and foci of the conic

$$x^2 - 2y^2 - 2x + 8y - 1 = 0$$

(b) Find the equation of the sphere of which the circle $xy + yz + zx = 0, x + y + z = 3$ is a great circle.

(c) Find the condition that the line

$\frac{1}{r} = A\cos\theta + B\sin\theta$ may touch the conic $\frac{1}{r} = 1 - e\cos\theta$.

(d) For what angle must the axes be turned to remove the term xy from $7x^2 + 4xy + 3y^2$.

(e) Find the equation of cone whose vertex is origin and the base curve is $x^2 + y^2 = 4, z = 2$.

6. Answer any *one* question : $1\times5=5$

(a) If r be the radius of the circle

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$,

$lx + my + nz = 0$ then prove that

$$\left(r^2 + d\right)\left(l^2 + m^2 + n^2\right) = \left(mw - nv\right)^2 + \left(nu - lw\right)^2 + \left(lv - mu\right)^2$$

and find the centre.

(b) Show that the feet of the normals from the point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the intersection of the ellipsoid and cone

$$\frac{\alpha a^2\left(b^2-c^2\right)}{x}+\frac{\beta b^2\left(c^2-a^2\right)}{y}+\frac{\gamma c^2\left(a^2-b^2\right)}{z}=0$$

7. Answer any *one* question : $10\times1=10$

(a) (i) Show that the plane $3x-2y-z=0$ cuts the cones $21x^2-4y^2-5z^2=0$ and $3yz-2zx+2xy=0$ in the same pair of perpendicular lines. 5

(ii) Find the equation of the cylinder, whose generators are parallel to the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and which passes through the conic $z=0, 3x^2+7y^2=12.$ 5

(b) (i) Find the locus of the point of intersection of the perpendicular generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1.$ 4

(ii) Reduce the equation

$$x^2+3y^2+3z^2-2xy-2yz-2zx+1=0$$

to its canonical form and determine the type of quadratic represented by it. 6

Unit - IV

8. Answer any *two* questions : $2\times2=4$

(a) Find the integrating factor of the differential equation

$$\left(2xy+3x^2y+6y^3\right)dx+\left(x^2+6y^2\right)dy=0$$

(b) Show that the general solution of the equation $\frac{dy}{dx}+Py=Q$ can be written in the form $y=k\left(u-v\right)+v$, where k is a constant and u and v are its two particular solutions.

(c) Solve : $\frac{dy}{dx}+y\cos x=xy^n$.

[Turn Over]

9. Answer any *one* question : $1\times5=5$

(a) The population of a country increases at the rate of proportional to the number of inhabitants. If the population doubles in 30 years, in how many years will it triple?

(b) Solve : $\left(px^2+y^2\right)\left(px+y\right)=\left(p+1\right)^2$

$\left[u=xy, v=x+y\right]$
