UG/4th Sem/MATH/H/19

2019

B.Sc. (Honours)

4th Semester Examination

MATHEMATICS

Paper - C10T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.

Unit - 1 (Group A)

1. Answer any ***three*** questions : 2×3

(a) Prove that in a ring R if a is an idempotent element then $1 - a$ is also idempotent. 2

(b) Define maximal ideal in a Ring. Give it's example. 2

(c) Define char R when char R is called the trivial ring. 2

[Turn Over]

(d) If a, b be two elements of a field F and $b \neq 0$, then prove that $a = 1$ if $(ab)^2 = ab^2 + bab - b^2$. 2

(e) If R is an integral domain, then $R[x]$ is also an integral domain. Where $R[x]$ is a power series ring. 2

2. Answer any *two* questions : 5×2

(a) Define divisors of zero in a ring. Show that the ring of matrices of the form $\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$ contains no divisor of zero if $a, b \in Q$ but contains divisor of zero if $a, b \in R$. 5

(b) Show that every field is an integral domain but the converse of the theorem is not necessarly true. 5

(c) Every ideal of the ring of integers $(Z, +, .)$ is a principal ideal. 5

Unit - 2 (Group B)

3. Answer any ***two*** questions : 2×2

(a) Prove that the rings $(Zn, +, .)$ and $(Z/(n), +, .)$ are isomorphic. 2

(b) Let $\{R, +, .\}$ and $\{R', +, .\}$ be two rings and $f : R \to R'$ be a homomorphism. Then prove that $f(-a) = -f(a), \forall a \in R$. 2

(c) Let $R = (Z, +, .)$; $R' = (2Z, +, .)$ and $\phi : R \to R'$ be defined by $\phi(x) = 2x,\ x \in Z$, show that ϕ is not a homomorphism. 2

4. Answer any ***two*** questions : 5×2

(a) State and prove 1st isomorphism theorem of Ring. 5

(b) Let I and J be two ideals of a ring R. Then $I + J$ and $I \cap J$ are also ideals and the factor ring $(I + J)$ and $I/(I \cap J)$ are isomorphic. 5

(c) Let $\{R, +, .\}$ and $\{R', +, .\}$ be two rings and $f : R \to R'$ be an isomorphism, then prove that

(i) if R be commutative then R' is also Commutative

(ii) if R contains unity then R' also containing unity.

(iii) if R be without divisor of zero then R' is also without divisor of zero. 5

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