2019

B.Sc.

2nd Semester Examination

MATHEMATICS (Honours)

Paper - C4T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Unit - I

[Marks - 22]

1. Answer any *one* question : 1×2

(a) Show that $\frac{dy}{dx} = 3y^{\frac{2}{3}}$, $y(0) = 0$ has more than one solution and indicate the possible reason.

(b) Let $W(y_1, y_2)$ be the Wrongkian of two linearly independent solutions y_1 and y_2 of the equation $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$ then prove that

$$W(y_1, y_2)P(x)=y_2\frac{d^2y_1}{dx^2}-y_1\frac{d^2y_2}{dx^2}.$$

2. Answer any *two* questions : 5×2

(a) Let r_1, r_2 be the roots of the indicial polynomial for the equation

$\frac{d^2y}{dx^2}+\frac{ady}{dx}+by=0$ where *a, b* are constants.

If $r_1 \neq r_2$ then show that two independent solutions are $e^{r_1 x}$ and $e^{r_2 x}$ respectively. 5

(b) Solve the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}=x+e^x \sin x$$

by the method of undetermine coefficient. 5

(c) Solve the differential equation

$$(2+3x)\frac{d^2y}{dx^2}+5(2+3x)\frac{dy}{dx}-3y=x^2+x+1,$$

$$-\frac{2}{3}<x<\alpha.$$

5

3. Answer any *one* equation : 10×1

(a) (i) A 2nd order linear differential equation of the form $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=F(x)$, where P, Q and F are continuous functions of x on $[a, b]$. Then by using the method of variation of parameters prove that the general solution of given ODE is given by

$$y(x)=Au(x)+Bv(x)\left\{-\int^{x}\frac{V(t)F(t)}{W(u,v)}dt\right\}u(x)+$$

$$\left\{\int^{x}\frac{u(t)F(t)}{W(u,v)}dt\right\}V(x),\ x\in[a,b]$$

[*Turn Over*]

where A and B are two arbitrary constants and $W(u,v) = uv' - vu'$. 5

(ii) Solve the diifferential equation

$\frac{d^2y}{dx^2} + a^2 y = \sec(ax)$ by the method of variation of parameters. 5

(b) (i) State and prove the super position principle for homogeneous linear differential equation. 4

(ii) Using the fact that $y = x^2$ is a solution of the equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$, $0 < x < \propto$ then find another independent solution. 4

(iii) If $y = x\cos x$ is a solution of an n-th order linear differential equation

$$\frac{d_y^n}{dx^n} + a_1 \frac{d_y^{x-1}}{dx^{n-1}} + \ldots + a_{n-1}\frac{dy}{dx} + a_n y = 0$$

with real constant coefficients, then find the least possible value of n. 2

Unit - II

[Marks - 13]

4. Answer any *four* questions : 2×4

(a) Show that

$$\left(yz+xyz\right)dx+\left(xy+xyz\right)dz+\left(zx+xyz\right)dy=0$$

is integrable.

(b) Let $y=\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ be a solution of the system of equations $\begin{pmatrix} y_1' \\ y_2' \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ a & b \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$. where $a,b\in R$

Then prove that every solution $y(x)\to 0$ as $x\to\propto$ if $a<0$ and $b<0$.

(c) Show that $f(t,x)=(3t+2x_1,\ x_1-x_2)$ on $S:\{|t|<\propto,\ \|x\|<\propto\}$ satisfying a Lipschitz condition where $x=(x_1,\ x_2)\in\mathbb{R}^2$.

(d) Find the first order simultaneous differential equations for the third order differential equation

$$\frac{d^3x}{dt^3}-6\frac{d^2x}{dt^2}+12\frac{dx}{dt}-8x=18e^{2t}.$$

(e) Solve : $\dfrac{dx}{3y-2z}=\dfrac{dy}{z-3x}=\dfrac{dz}{2x-y}$.

(f) Write fundamental matrix for homogeneous system of linear equation given below

$\dot{X}(t)=A(t)X(t)$ where

$$X(t)=\begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix},\ t\in[a,\ b].$$

5. Answer any *one* question : 5×1=5

(a) Find the fundamental matrix and the solution $X(t)$ such that $X(0)=\begin{pmatrix} 1 \\ 6 \end{pmatrix}$ for the system

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 0 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$ 5

(b) (i) Solve :

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)}$$ 2½

(ii) Solve :

$$(yz+2x)dx + (zx-2z)dy + (xy-2y)dz = 0$$
2½

Unit - III

[Marks - 9]

6. Answer any *two* questions : 2×2

(a) Find the equilibrium point of the system of differential equations

$$\dot{x} = e^{x-1} - 1 \text{ and } \dot{y} = ye^x .$$

[*Turn Over*]

(b) Show that $x=0$ is the regular singular point of the differential equation

$$2x^2\frac{d^2y}{dx^2}+7x(x+1)\frac{dy}{dx}-3y=0$$

(c) If $\sum_{m=0}^{\infty} c_m x^{r+m}$ is assumed to be a solution of

$x^2y''-xy'-3\left(1+x^2\right)y=0$ then find the values of r.

7. Answer any *one* question : 5×1

(a) Find a power series solution of the equation

$$\left(x^2-1\right)\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+xy=0.$$

Given that $y(0)=4$ and $\left.\frac{dy}{dx}\right|_{x=0}=6$. 5

(b) (i) Find the phase curve of the dynamical system of equations $\dot{x}=2x-y$ and $\dot{y}=-4y$. Also describe the nature of stationary point.

(ii) Determine the steady state and their stability of the differential equation $\frac{dx}{dt} = x^2 - 5x + 6$.

3+2=5

Unit - IV

[Marks - 16]

8. Answer any *three* questions : 2×3

(a) If the vector $\vec{a}$ and $\vec{c}$ are perpendicular to each other then show that the vectors

$\vec{a} \times (\vec{b} \times \vec{c})$ and $(\vec{a} \times \vec{b}) \times \vec{c}$ are perpendicular to each other.

(b) Find the value of x such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $x\hat{i} - 4\hat{j} + 5\hat{k}$ are Coplanar.

(c) If $\vec{a} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$ and

$\vec{b} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$ then show that

$\frac{d}{dt}(\vec{a}.\vec{b}) = -6$ at $t = 1$.

[Turn Over]

(d) Evaluate

$$\int_0^1 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2} \right) dt \text{ where } \vec{r} = t^3\hat{i} + 2t^2\hat{j} + 3t\hat{k}.$$

(e) A necessary condition for the vector $\vec{c}(t)$ to be constant is $\frac{d\vec{c}}{dt} = \vec{0}$. Prove it.

9. Answer any *one* question : 10×1

(a) (i) A necessary and sufficient condition that a proper vector $\vec{u}$ has a constant length is that $\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$. 5

(ii) If $\frac{d\vec{a}}{dt} = \vec{r} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{r} \times \vec{b}$ then show that $\frac{d}{dt}\left(\vec{a} \times \vec{b}\right) = \vec{r} \times \left(\vec{a} \times \vec{b}\right)$ where $\vec{r}$ is a constant vector and $\vec{a}, \vec{b}$ are vector functions of a scalar variable t. 5

(b) (i) Four points whose position vectors are $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are Coplanar if and only if

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \left[\vec{b}\,\vec{c}\,\vec{d}\right] + \left[\vec{c}\,\vec{a}\,\vec{d}\right] + \left[\vec{a}\,\vec{b}\,\vec{d}\right] \quad 5$$

(ii) If $\frac{d^2\vec{r}}{dt^2} = 6t\hat{i} - 24t^2\hat{j} + 4\sin t\,\hat{k}$ and if

$\vec{r} = 2\hat{i} + \hat{j}$ and $\frac{d\vec{r}}{dt} = -\hat{i} - 3\hat{k}$ when $t = 0$,

then show that

$$\vec{r} = \left(t^3 - t + 2\right)\hat{i} + \left(1 - 2t^4\right)\hat{j} + \left(t - 4\sin t\right)\hat{k}.$$

5