2019

B.Sc.

2nd Semester Examination

MATHEMATICS (Honours)

Paper - C3T

(Real Analysis)

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Unit - I

(Real number system and sets in R)

[Marks - 24]

1. Answer any *two* questions : 2×2=4

(a) Define interior of a set S. Find the interior of the set

$$S = \{x \in R : 1 < x < 3\}$$

[Turn Over]

(b) Give an example of an open cover of the set $(0, 1]$, which does not have a finite subcover. 2

(c) Prove that the number $\sqrt{2}+\sqrt{3}$ is irrational.

2. Answer any *two* questions : 5×2=10

(a) If $S=\left\{(-1)^m+\frac{1}{n};\ m,n\in N\right\}$, then show that 1 and –1 are limit points of S. Find the derived set S' of S. 2+2+1

(b) Prove that union of an enumerable number of enumerable sets is enumerable.

(c) Show that the intersection of finite collection of open sets is an open set. Show by examples that intersection of arbitrary collection of open sets may not be open. 3+2

3. Answer any *one* question : 10×1=10

(a) (i) Let S be a closed and bounded subset of R. Then prove that every open cover of S has a finite subcover. 6

(ii) State and prove Arcimedean property of real numbers. 1+3

(b) Define closed set. If S be a non-empty bounded subset of R, then prove that Sup S and Inf S belong to S. Find Sup S and Inf S of the set

$$S=\left\{\frac{(-1)^n}{n}+\sin\frac{n\pi}{4};\ n\in N\right\}$$ 1+4+2

Unit - II

(Real Sequence)

[Marks - 18]

4. Answer any *four* questions : 2×4=8

(a) Prove that $\lim\limits_{n\to\infty}\left(2^n+3^n\right)^{\frac{1}{n}}=3$.

(b) Find $\underline{\lim}\, u_n$ and $\overline{\lim}\, u_n$ where $u_n=\frac{n}{2}-\left[\frac{n}{2}\right]$ where $[K]$ is the greatest integer not greater than K. 2

(c) State Bolzans-Weierstrass theorem for sequences. Give an example of a bounded sequence. Give

[Turn Over]

an example of a bounded sequence having exactly two limit points.

(d) If a sequence $\{a_n\}$ converges to l, then prove that the sequence $\{|a_n|\}$ converges to $|l|$.

(e) Assuming that the sequence $\left(1+\frac{1}{n}\right)^n$ converges to e, prove that the sequence $\left(1+\frac{1}{2n}\right)^n$ converges to $\sqrt{e}$.

(f) If $\{u_n\}$ be a bounded sequence and $\lim_{n\to\infty} v_n = 0$ then prove that $\lim_{n\to\infty} u_n v_n = 0$. 2

5. Answer any *one* question : 10×1=10

(a) (i) Prove that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots+\frac{1}{\sqrt{n^2+n}}\right]=1$$

5

(ii) Prove that a monotone increasing sequence, if bounded above, is convergent and it converges to the least upper bound. 5

(b) (i) If $\{x_n\}$ converges to l then prove that the sequence $\left\{\frac{x_1 + x_2 + ... + x_n}{n}\right\}$ converges to l. 5

(ii) A sequence $\{x_n\}$ defined by

$x_{n+2} = \frac{1}{2}(x_{n+1} + x_n)$ for $n \geq 1$ and

$x_1 = \frac{1}{3}$, $x_2 = \frac{1}{2}$ prove that the sequence

$\{x_n\}$ converges to $\frac{4}{9}$. 5

[Turn Over]

Unit - III

(Infinite Series)

[Marks - 18]

6. Answer any *four* questions : $4\times2=8$

(a) Using Integral test, examine the convergence of

$$\sum\frac{1}{n\log n}.$$ 2

(b) Using D'Alembert's Ratio Test to examine the convergence of the series $\sum_{n=1}^{\infty}\frac{4^n}{\angle n}$. 2

(c) Using comparison test show that the series

$$\sum_{n=1}^{\infty}\frac{\sin(n\alpha)}{n^2},\ \alpha>0 \text{ is convergent.}$$

(d) State Gauss test for the convergence of a series of positive terms. 2

(e) When a series is said to be conditionally convergent. Give an example of a conditionally convergent series.

(f) If $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ be both convergent prove that the series $\sum_{n=1}^{\infty} x_n y_n$ is absolutely convergent.

7. Answer any *two* questions : 5×2=10

(a) Discuss the convergence of the following series

$$1+\frac{\alpha.\beta}{1.\gamma}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}x^2$$

$$+\frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{1.2.3.\gamma(\gamma+1)(\gamma+2)}x^3+\ldots$$ 5

(b) Investigatge the convergence of the series

$$1-\frac{2!}{2^2}+\frac{3!}{3^3}-\frac{4!}{4^4}+\frac{5!}{5^5}-\ldots$$ 5

(c) Let $\sum u_n$ be a series of arbitrary terms and $p_n=\frac{u_n+|u_n|}{2}$ and $q_n=\frac{u_n-|u_n|}{2}$. Prove that

(i) both $\sum p_n$ and $\sum q_n$ are convergent if $\sum u_n$ is absolutely convergent and (ii) both $\sum p_n$ and $\sum q_n$ are divergent if $\sum u_n$ is conditionally convergent. 3+2
