2018

CBCS

1st Semester

MATHEMATICS

PAPER—C2T

(Honours)

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Algebra

Unit - I

Classical Algebra

1. Answer any *one* quesiton : 1×2

(a) If the complex numbers z_1, z_2 and z_3 represent the three points P, Q, R and be such that

$$lz_1 + mz_2 + nz_3 = 0$$

Where $l + m + n = 0$, then show that the points P, Q, R lie on a straight line.

(Turn Over)

(b) Apply Descarte's rule of signs to ascertain the minimum number of complex roots of the equation $x^6 - 3x^2 - 2x - 3 = 0$.

2. Answer any *two* questions : 2×5

(a) Prove that :

$$x^n + 1 = \prod_{k=0}^{\frac{n-2}{2}} \left[x^2 - 2x\cos\frac{(2k+1)\pi}{n} + 1 \right],$$

if n be an even positive integer. Deduce the

$$\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16} = \frac{1}{8\sqrt{2}}.$$ 4+1

(b) Solve the equation

$x^3 - 15x^2 - 33x + 847 = 0$

by Cardan's method.

(c) State and Prove Cauchy Schwarz's inequality.

3. Answer any *one* question : 1×10

(a) (i) Show that the solution of the equation

$$(1+x)^n - (1-x)^n = 0 \text{ are } x = i\tan\frac{\pi r}{n},$$

where $r = 0, 1, 2, \ldots, n-1$, if n be odd

$= 0, 1, 2, \ldots, \frac{n}{2}-1, \frac{n}{2}+1, \ldots, n-1$, if n be even.

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(ii) If x, y, z be positive and $x + y + z = 1$, then show that $8xyz \le (1-x)(1-y)(1-z) \le \frac{8}{27}$. Show that

$$3x(3x+1)^2 > 4\left(\underline{|3n}\right)^{\frac{1}{n}}$$

where n be a positive integer (>1). 3 + 2

(b) (i) Solve the equation

$$x^4 - 12x^3 + 47x^2 - 72x + 36 = 0$$

given that the product of two of the roots is equal to the product of the other two. 4

(ii) State Descartes' rule of signs. Obtain the equation whose roots exceed the roots of the equation $x^4 + 3x^2 + 8x + 3 = 0$ by 1.

Use Descartes' rule of signs to both the equations to find the exact number of real and complex roots of the given equation. 1 + 2 + 3

Unit—II

Sets and Integers

4. Answer any *five* questions : 5×2

(a) Find $f \circ g$, if $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = |x| + x,\ x \in \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ is defined by $g(x) = |x| - x,\ x \in \mathbb{R}$.

(b) Let $f : A \to B$ and $P \subseteq A$. Prove that $P \subseteq f^{-1}f(P)$.

(c) Let $f : A \to B$. If $S \subset A$, then show that $S \subset f^{-1}[f(S)]$. If further f be one-one and onto, then prove that $f^{-1}[f(s)] = S$.

(d) Find integers u and v statisfying $52u - 91v = 78$.

(e) Using the principle of induction, prove that $2.7^n + 3.5^n - 5$ is divisible by 24 for $n \in N$.

(f) Find the remainder when $1! + 2! + 3! + \ldots.. + 50!$ is divided by 15.

(g) If a is prime to b and a is prime to c then prove that a is prime to bc.

(h) Find the units digit in 7^{99}.

5. Answer any *one* question : 1×5

(a) (i) Use division algorithm to prove that the square of an odd integer is of the form $(8k + 1)$, where k is an integer.

(ii) Use Euclidean algorithm to find integers u and v such that $gcd(72, 120) = 72u + 120v$. 3+2

(b) Define equivalence relation. A relation ρ is defined on $\mathbb{N} \times \mathbb{N}$ by "$(a, b)\rho(c, d)$ if and only if $ad = bc$" for $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$. Show that ρ is an equivalence relation. 1+4

Unit—III

System of Linean Equations

6. Answer any *two* questions : 2×2

(a) Find a row echelon matrix which is row equivalent to

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}.$$

(Turn Over)

(b) Show that the planes $2x - y + z = 5$, $x + 2y + 4z = 7$, $5x + 3y - z = 0$ are concurrent.

(c) Let x, y, z be elements of a vector space V over F and let $a, b \in F$. Show that x, y, z are linearly dependent, if $(x + ay + bz)$, y, z be linearly dependent.

7. Answer any *one* question : 1×5

(a) Investigate, for what values of α and μ, the following equations 5

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have i) No solution

ii) a unique solution

and iii) an infinite number of solutions.

(b) i) Obtain the fully row reduced normal form of the matrix : 3 + 2

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

ii) Find the value of k, such that the following system of linear equation is consistent :

$$2x + y - z = 12,\ x - y - 2z = -3,\ 3y + 3z = K.$$

Unit—IV

Linear Transformation and Eigen Values

8. Answer any *two* questions : 2×2

(a) A and B are any two 2×2 matrices and E is the corresponding unit matrix. Show that $AB - BA = E$ cannot hold under any circumstances.

(b) If λ be an eigen value of a non-singular matrix A, then prove that λ^{-1} is an eigen value of A^{-1}.

(c) If $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ then show that $A^2 = A^{-1}$.

9. Answer any *one* question : 1×10

(a) i) Let $S = \{(x, y, z, w) \in \mathbb{R}^4 : x + 2y - z = 0, 2x + y + w = 0\}$ Prove that S is a subspace of the real vector space $\mathbb{R}^4$. Also find the basis of S and the dimension of S. 2+2+1

ii) A is a 3×3 real matrix having the eigen values 2, 3, 1. If $\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}$ are the eigen vectors of A corresponding to the eigen values 2, 3, 1 respectively. Find the matrix A. 2+2+1

(b) i) Prove that the eigen values of a real skew-symmetric matrix are purely imaginary or zero. 5

ii) Let V be a vector space over a field F and let $\alpha, \beta \in v$. Then prove that the set

$w = \{c\alpha + d\beta : c \in F, d \in F\}$ from a subspace of V.

If $\alpha = (1, 2, 3)$, $\beta = (3, 1, 0)$ and $\gamma = (2, 1, 3)$ then examine for $\gamma \in w$ or not. 5

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