

2017

MATHEMATICS

[Honours]

(CBCS)

[First Semester]

PAPER – C1T

Full Marks : 60

Time : 3 hours

The figures in the right hand margin indicate marks

UNIT – I

(Calculus – I)

1. Answer any *three* questions :

2 × 3

(a) What do you mean by asymptote ? Does asymptote exist for every curve ?

(b) Your answer find the value of

$$y = \lim_{x \rightarrow 1} x^{\frac{1}{x-1}}$$

$$\log y = \lim_{x \rightarrow 1} \frac{\log x}{x-1}$$

$$= \frac{1}{1} = 1$$

(Turn Over)

$$\therefore y = e^1 = e$$

(c) Define point of inflection of a curve.

(d) Find the envelope of the straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$

where the parameters a and b are connected by the relation $ab = c^2$.

(e) Write the Leibnitz theorem successive derivatives up to 4th order.

2. Answer any one question :

10 × 1

(a) (i) If s be the length of the arc $3ay^2 = x(x-a)^2$ measured from the origin to any point (x, y) , show that

$$s = \int_0^x \sqrt{1 + \frac{dy}{dx}} dx$$

$$3s^2 = 4x^2 + 3y^2.$$

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(ii) Show that the curve

$$y = 3x^5 - 40x^3 + 3x - 20$$

is concave upwards for $-2 < x < 0$ and $2 < x < \infty$ but convex upwards for $-\infty < x < -2$ and $0 < x < 2$. Also show that $x = -2, 0, 2$ are its points of inflexion.

2 + 2 + 1

(3)

(b) (i) Trace the curve :

$$x^2 y^2 = a(y^2 - x^2) \quad 5$$

(ii) If α, β be the roots of the equation $ax^2 + bx + c = 0$ then show that

$$\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2} = \frac{1}{2} a^2 (\alpha - \beta)^2 \quad 5$$

(Calculus - II)

3. Answer any two questions : 2 x 2

(a) If

$$I_n = \int_0^{\pi/4} \tan^n x dx,$$

n being a positive integer greater than 1, then prove that

$$I_n = \frac{1}{n-1} - I_{n-2}.$$

(b) The volume of the solid generated by the revolution of the curve $y = \frac{1}{x}$, bounded by

$$y = \dots \quad (4)$$

$$\int y^2 dx = \int \sqrt{x} \cdot \frac{1}{2} dx = \left[\frac{2}{3} x^{3/2} \right]$$

$y = 0, x = 2, x = b$ ($0 < b < 2$) about x -axis is 3. Find the value of b .

(c) What is the formula for finding area of the curve $y = \psi(t), x = \phi(t)$, where t is the parameter?

4. Answer any two questions:

$$\int y dx = \int \psi(t) \phi'(t) dt$$

(a) If

$$I_{m,n} = \int_0^{\pi/2} \cos mx \sin nx dx,$$

then show that

$$I_{m,n} = \frac{1}{m+n} + \frac{1}{m-n} I_{m-1,n-1}.$$

Also, deduce that

$$I_{m,n} = \frac{1}{2^{m+1}} \left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \dots + \frac{2^m}{m} \right].$$

(b) Find the area of the surface generated by revolving the curves $x = \cos t, y = 2 + \sin t, 0 \leq t \leq 2\pi$ about x -axis.

$\int y^2 dx$

$$\frac{(y-2)^2}{2} = 1 - a^2$$

$$\therefore a^2 + (y-2)^2 = 1$$

(5)

(c) Find the arc length parameter along the curve

$$C: \vec{r}(t) = (1+2t)\hat{i} + (1+3t)\hat{j} + 6(1-t)\hat{k}$$

from the point, where $t = 0$.

UNIT - III

(Geometry)

5. Answer any *three* questions :

2 × 3

(a) Determine the type of the conic

$$8x^2 + 10xy + 3y^2 + 22x + 14y + 15 = 0$$

(b) Show the plane $z - 1 = 0$ which intersects the ellipsoid

$$\frac{x^2}{48} + \frac{y^2}{12} + \frac{z^2}{4} = 1$$

is an ellipse. Determine semi-axes.

(c) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 25$, $x + 2y - z + 2 = 0$ and the point $(1, 1, 1)$.

(d) Find the equation of the cylinder where generators are parallel to the line $x = -\frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + y^2 = 1, z = 3$.

(e) Find the equation of a right circular cone whose axis is $\frac{x}{1} = \frac{y}{0} = \frac{z}{-2}$ and radius equal to 7.

6. Answer any *one* question : 5 × 1

(a) Find the polar equation of the tangent to the circle $r = 2d \cos\theta$ at the point whose vectorial angle is θ_1 .

(b) Find the equations of the generating lines of the hyperboloid of one sheet

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$$

which passes through the point $(2, 3, -4)$.

7. Answer any *one* question : 10 × 1

(a) (i) Find the locus of a luminous point, if the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ casts a circular shadow on the plane $z = 0$ 5

(ii) Reduce the equation

$$4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$$

to canonical form. 5

(b) (i) Prove that the locus of the foot of the perpendicular from a focus of the conic

$\frac{l}{r} = 1 - e \cos \theta$ on a tangent to it, is given by

$$r^2(1 - e^2) - 2ler \cos \theta - l^2 = 0 \quad 5$$

(ii) Prove that the axes of sections of the conicoid $ax^2 + by^2 + cz^2 = 1$ which pass through the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ lie on the cone

$$\frac{(b-c)}{x}(mz - ny) + \frac{(c-a)}{y}(nx - lz) + \frac{(a-b)}{z}(ly - mx) = 0. \quad 5$$

UNIT - IV

(Differential Equation)

8. Answer any *two* questions : 2 × 2

(a) Find the integrating factor of the following differential equation

$$\frac{dx}{dy} + \frac{xy}{1-y^2} - y\sqrt{x} = 0$$

(b) Reduce the equation

$$\sin y \frac{dy}{dx} = \cos x (2 \cos y - \sin^2 x)$$

into a linear equation.

(c) Show that the equation

$$M(x, y)dx + N(x, y)dy = 0$$

will be exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

9. Answer any *one* question : 5 × 1

(a) (i) Show that the substitution $z = ax + by + c$ changes

$$y' = f(ax+by+c)$$

into an equation with separable variables, and apply this method to solve the equation

$$y' = \sin^2(x-y+1)$$

(ii) Reduce the equation

$$(2x^2-1)\left(\frac{dy}{dx}\right)^2 + (x^2+y^2+2xy+2)\frac{dy}{dx} + 2y^2+1=0$$

to Clairaut's form by the substitution $x+y=u$ and $xy-1=v$, hence solve the equation.

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(b) (i) Show that if y_1 and y_2 be solutions of the equation

$$\frac{dy}{dx} + Py = Q$$

where P and Q are functions of x alone,

and $y_2 = y_1 z$ then $z = 1 + ae^{-\int \frac{Q}{y_1} dx}$

(ii) Show that the solution of $\frac{dy}{dx} + py = Q$
can also be written in the form

$$y = \frac{Q}{P} - e^{\int P dx} \left[c + \int e^{\int P dx} d\left(\frac{Q}{P}\right) \right] \quad 5$$

$$\frac{d}{dx} (y \cdot e^{\int p dx}) = Q \cdot e^{\int p dx}$$

$$y \cdot e^{\int p dx} = \int Q \cdot e^{\int p dx} dx + C$$

$$\therefore y = e^{-\int p dx} \left[\int Q e^{\int p dx} dx + C \right]$$